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XtraEdge for IIT-JEE

Volume - 7 Issue - 1

July, 2011 (Monthly Magazine)

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WORRY IS A MISUSE OF IMAGINATION.

Editorial

Dear Students,

"People with integrity do what they say they are going to do. Others have excuses."

Rudyard Kipling, the celebrated English author and poet, once said, "We have forty million reasons for failure, but not a single excuse." Yet today we are literally inundated with a tidal wave of excuses from every direction. In fact, it seems everyone has a reason, explanation or justification for not doing what they were supposed to do.

Why do so many of us crank out one excuse after another for virtually everything we fail to do? Well, for starters, excuses are easy. In fact, they're way too easy. After all, making excuses doesn't require any effort or commitment on our part. All we have to do is toss out excuse after excuse and we feel we're off the hook, since the best excuses always absolve us of any personal responsibility whatsoever.

While getting in the habit of making of excuses is easy, excuse making doesn't get any of us anywhere close to where we want to go in life. Sooner or later all of our years of excuses eventually catch up with us. Before we realize it, the best of life has slipped away in a lazy, hazy, crazy blur of excuses.

Ninety-nine percent of the failures come from people who have the habit of making excuses. Hold yourself responsible for a higher standard than anybody else expects of you, never excuse yourself.

The person who really wants to do something invariably finds a way to get it done. And for those who don't want to do something; well, one excuse is just as good as another I suppose. What it all boils down to is simply this: what kind of person do you really want to be? Do you want to make excuses - or make something happen instead?

It's time to turn all of your excuses loose once and for all. Each and every time you fall short, pick yourself up, learn from your mistakes and immediately get going again. No complaining, no explaining and absolutely no excuses allowed. You will find that the minute you stop making excuses and start finding a way to get the job done, you'll start making your life everything it could be and should be... and so much more.

Get rid of the excuses and you can get anywhere you've ever dreamed of going.
Presenting forever positive ideas to your success.

Yours truly

Pramod Maheshwari,
B.Tech., IIT Delhi

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NEXT MONTHS ATTRACTIONS

- Much more IIT-JEE News.
- Know IIT-JEE With 15 Best Questions of IIT-JEE
- Challenging Problems in Physics,, Chemistry & Maths
- Key Concepts & Problem Solving strategy for IIT-JEE.
- Xtra Edge Test Series for JEE- 2012 & 2013

Success Tips for the Months

- The difference between a successful person and others is not a lack of strength, not a lack of knowledge, but rather a lack of will.
- Footprints on the sands of time are not made by sitting down.
- To succeed, we must first believe that we can.
- The secret of joy in work is contained in one word - excellence. To know how to do something well is to enjoy it.
- Six essential qualities that are the key to success: Sincerity, personal integrity, humility, courtesy, wisdom, charity.
- Continuous efforts - not strength or intelligence - is the key to unlocking our potential.
- We can do anything we want to do if we stick to it long enough.
- The path to success is to take massive, determined action.

CONTENTS

Now get Online access to IIT lectures, courses

India's premier technical institute, the IITs, will now make all its engineering courses and lectures available for downloads to students, even on their mobile phones.

This is a part of an initiative by the National Programme on Technology Enhanced Learning (NPTEL). It is the country's largest technical knowledge dissemination programme.

So if you want an access to the lectures given by professors of IIT, IISc and NITs, you can do it from the NPTEL website www.nptel.iitm.ac.in and from YouTube www.youtube.com/iit. Students can get lectures in electrical, mechanical, civil and computer science engineering, and in both in web and video format.

During its first phase, each course will offer around 40 hours of lectures delivered by the country's best teachers from different IITs and the IISc

No more traffic jam as IITian designs compact electric car

In an attempt to fight the menace of traffic jams and decreasing road space, a student of master of design programme of IIT-Kanpur has come up with a genius sample of a compact electric car which can be extended at the rear end to make a second seat. Targeting office goers who drive alone, the car has a single seat and is apt for those who do not use the entire seating capacity of the vehicle. Just one person in the car, these officer goers take up most of the road space, leading to traffic congestion.

The car, designed by Kumar Sudeepta, an MDes student of IIT-Kanpur, has been kept on display at the annual design exposition.

Christening the car as Growing Electric Vehicle (GeV), Sudeepta describes his car as front wheel-powered battery-propelled four-wheeler, which is designed on the lines of green urban transportation. The vehicle will surely address the problems of parking, traffic congestion and wastage of road space.

The main idea of GeV is to reduce the size of the vehicle, so that the cost of production of the vehicle is reduced and is pocket-friendly for the masses.

Made from fibre reinforced plastic, the body is light but at the same time has the strength and toughness of an ordinary car. The battery is situated in the front as it is a front wheel powered electric vehicle. The space created in the cabin near the rear seat can also be used to place additional luggage.

Explaining his invention to *The Times of India*, Sudeepta said, "GeV is a compact vehicle meant to carry one person on a normal course. It has a unique feature to extend at the rear and generate a second seat from beneath the driving seat inside the cabin so that a second passenger can also join in if need arises. The rear seat lies beneath the driving seat. When the cabin is extended it flushes out of the driving seat and gives enough space for a second rider."

Computer based JEE Exam?

Computer based entrance exams are becoming more popular among students and as well in exam conducting authorities. Last year Common Admission Test went online and All India Engineering Entrance Exam too followed the same path. Now, it is time for JEE to go online, JEE chairman decided to follow the suit. This prestigious entrance exam will soon become computer-based .

As the number of the IIT aspirants increasing, Joint admission board member said that it would take almost three years to develop a question bank before the JEE could be offered on computer mode.

IIT heads expressed their concern over the move and said that they would not be able to provide the question bank for JEE exam soon, as the JEE is also conducting in Singapore, and time differences would hit the testing process and time schedule.

IITs to help technical institutes

IITs may soon help to boost the standard of technical and engineering institutes which are often in limelight for offering poor academic standard to students.

The 40-year-old Institutes of Technology Act is being amended so that IITs bear additional responsibility of supporting and collaborating with technical education institutions and give advice to state governments on technological problems within the zone they are located.

The Institutes of Technology (Amendment) Bill, 2010 has made it mandatory for all 15 IITs to provide training, facilitate study visits, share laboratory and other resources with technical education institutions in their zones.

India, France seal IIT-Rajasthan plan

Taking their ties to the next level, India and France have recently signed two memorandums of understanding (MoUs) on higher education, and also decided to take forward 2009 `Plan of Action for IIT-Rajasthan' by setting up a French consortium that will help the institute gain expertise in areas like solar energy, aerospace, quantum computing, health technology and other fields.

The other MoU stresses on academic collaboration with seven IITs and ParisTech - Paris Institute of Science and Technology. This agreement seeks to promote institutional exchange, students exchange, short courses, educational programmes and joint research and exchange of information in teaching and student development.

IIT-B explains power of Homeopathy

Six months after the British Medical Association lashed homeopathy as witchcraft with no scientific basis, IIT scientists have said the sweet white pills work on the principle of nanotechnology.

In a latest research conducted by IIT, which is published in the latest issue of Homeopathy, a peer-reviewed journal from medical publishing firm Elsevier, it was said that Homeopathic pills contain naturally occurring metals such as gold, copper and iron retain their potency even when diluted to a nanometre or one-billionth of a metre.

These results were found after students of IIT-B chemical engineering department bought homeopathic pills from nearby shops, prepared highly diluted solutions and checked these under powerful electron microscopes to find nanoparticles of the original metal.

The research says that homeopathy has been a mystery for modern medicine. Its practitioners maintained that homeopathic pills got more potent on dilution, but they could never explain the mechanism scientifically enough for the modern scientist.

Students of IIT Bombay have launched their own campus web based radio station.

The students were inspired by leading universities like MIT, Harvard and Yale that have their own successful on campus radio stations. Sajid Shariff, the general secretary of cultural affairs, IIT Bombay, says that they intend to provide the students through their LAN based radio station with a platform to voice their opinions, take part in debates and allow them to be more informed. The shows will include campus, city, national and international news, music, reviews of websites, information on books to read, gadgets, movies to watch, places and restaurants to hang out and extra-curricular activities. This is the first time a university in India has taken such an initiative.

IIT Kanpur's nano satellite may be launched by June-end

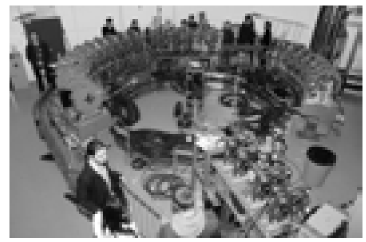
KANPUR: IIT Kanpur's indigenously built nano satellite 'Jugnu' may be successfully launched by the end of June this year from Sriharikota if everything goes according to plans.

Scientists from the Indian Space Research Organisation (Isro) had visited the institute last month and had given a green signal to the engineering model. The flight model is being assembled in Bengaluru.

IIT-K director Sanjay Govind Dhande today said that over 60 students and faculty members of the institute worked hard to built 'Jugnu', which weighs 3 kg.

Particle Accelerators Like This One Could Bring Safer Nuclear Power to Neighborhoods

The Electron Model of Many Applications, EMMA,



EMMA Installation Science and Technology Facilities Council-UK

A particle accelerator in the English countryside could be a harbinger of a safer, cleaner future of energy. Specifically, nuclear energy, but not the type that has brought havoc in Japan and controversy throughout Europe and the U.S. It would be based on thorium, a radioactive element that is much more abundant, and much more safe, than traditional sources of nuclear power.

Thorium reactors would not melt down, in part because they require an external input to produce fission. Thorium atoms would release energy when bombarded by high-energy neutrons, such as the type supplied in a particle accelerator.

- EMMA operates at around 20 MeV, or 20 million electronvolts, a paltry amount for an atom accelerator. The Tevatron, for instance, accelerates particles to 1 tera-electron volts. The Large Hadron Collider is designed to speed them to 7 TeV. But thorium reactors would not need such high energies to initiate fusion.

Success Story

This article contains storys/interviews of persons who succeed after graduation from different IITs

Mr. Avinash Chander

B-Tech (IIT Delhi)

Mr. Avinash Chander obtained B.Tech degree from Indian Institute of Technology (IIT Delhi) in Electrical Engineering in 1972 and M.S. (Spatial Information Tech.), JNTU, Hyderabad. . Mr. Avinash Chander, Distinguished Scientist is the Director of Advanced Systems Laboratory, Hyderabad and an eminent scientist in the field of Missiles.

Mr. Chander joined DRDO in August 1972. A leader in missile technology, he is the Chief Designer of Long Range Missiles System. He has the unique achievement of delivering and deploying three weapon systems viz., A1, A2 and A3. He is currently the Programme Director for the Agni Missile Programme. His latest achievement is the successful development of 3000 kms range Agni-III Missile which is under induction.

AWARDS & HONOURS:

- DRDO Scientist of the year 1989 - Awarded by Dr. PV. Narasimha Rao, Hon'ble Prime Minister of India.
- Astronautical Society of India Award for the year 1997 in the field of Rocketry -
- Awarded by Dr. R. Chidambaram, Principal Scientific Advisor to Govt. of India.
- AGNI Self-Reliance Award in 1999 - Awarded by Sri Atal Behari Vajpayee, Hon'ble Prime Minister of India.
- Dr. Biren Roy Space Science Award in 2000.
- DRDO Award for the year 2007 for Path Breaking Research /Outstanding Technology Development - Awarded by Hon'ble Dr. Manmohan Singh, Prime Minister of India_
- Outstanding Technologist Award 2008 by Punjab Technical University, Jalandhar._
- DRDO Technology Leadership Award 2008 - Awarded by Dr. A.K. Anthony, Hon'ble Defence Minister of India.
- Fellow, Indian National Academy of Engineers & Convenor, Sub-Committee of Aerospace Section.
- Fellow, System Society of India.
- Fellow, Andhra Pradesh Academy of Sciences.
- Fellow & Vice-President, Astronautical Society of India.
- Chairman–INSARM & ISAMPE, Hyderabad Chapter.

Mr. Rahul Bhattacharjee

M-Tech

Rahul Bhattacharjee, Topper, M Tech dual degree in Civil Engineering.

Confident and ready to take on life's new challenges, Rahul, is excited about stint at Massachusetts Institute of Technology. As his parents were both IIT-ians, Rahul made them proud by keeping the IIT tradition intact and passed out with flying colours. Rahul, who hails from Kolkata has settled in Mumbai.

IIT experience

Studying at IIT is an amazing experience because it is a pool of brains from diverse backgrounds. We get to meet and interact with people from different backgrounds: this helps instill in us many good qualities. It teaches you more and more. The qualities in several people appeal to you and our interaction with them help us develop into better individuals.

Your intellect matters too and, of course, a bit of luck!

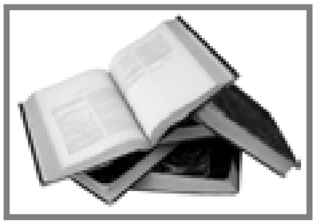
Mantra for success

I had my father as a role model. I also put in a lot of hard work: about 8 to 9 hours everyday after the 10th standard.

Advice to IIT aspirants

Work hard. Keep your focus, be good at handling problems. Put in a lot of effort. Start right form your 10th standard.

The most important things is not just getting an admission in an IIT, but you learning to utilise the time and resources at the IIT in the best possible manner.

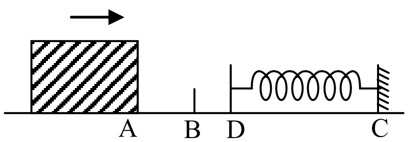


KNOW IIT-JEE

By Previous Exam Questions

PHYSICS

1. A 0.5 kg block slides from the point A (see fig.) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0.2 respectively. If the distances AB and BD are 2m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely. (Take $g = 10\ m/s^2$) **[IIT-1983]**



Sol. K.E. of block = work against friction + P.E. of spring

$\frac{1}{2}mv^2 = \mu_k\, mg\,(2.14 + x) + \frac{1}{2}kx^2$

$\frac{1}{2} \times 0.5 \times 3^2 = 0.2 \times 0.5 \times 9.8(2.14 + x) + \frac{1}{2}\, 2 \times x^2$

$2.14 + x + x^2 = 2.25$

$\therefore x^2 + x - 0.11 = 0$

On solving we get $x = -\frac{11}{10}$

or $x = \frac{1}{10} = 0.1$ (valid answer)

Here the body stops momentarily.

Restoring force at $y = kx = 2 \times 0.1 = 0.2$ N

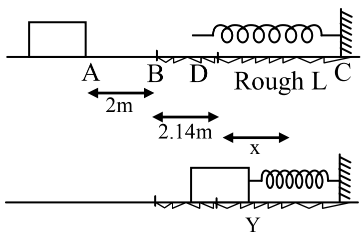
Frictional force at

$y = \mu_s\, mg \times x = 0.22 \times 0.5 \times 9.8 = 1.078$ N

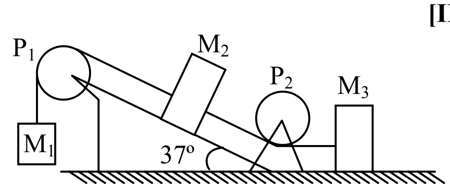
Since friction force > Restoring force the body will stop here.

$\therefore$ The total distance travelled

= AB + BD + DY = 2 + 2.14 + 0.1 = 4.24 m.



2. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and friction less pulleys P_1 and P_2 as shown in fig. The masses move such the portion of the string between P_1 and P_2 in parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37º with the horizontal. **[IIT-1981]**



If the mass M_1 moves downwards with a uniform velocity, find

(a) the mass of M_1

(b) The tension in the horizontal portion of the string ($g = 9.8\ m/sec^2$, $\sin 37º \approx 3/5$)

Sol. **(a)** Applying Fnet = ma on M_1 we get

$T - m_1 \cdot g = M_1 \times 0 = 0 \;\Rightarrow\; T = M_1g$ ...(i)

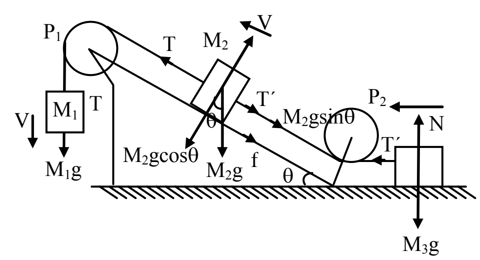
Applying Fnet = Ma on M_2 we get

$T - (T' + M_2g\sin\theta - f) = M_2 \times a$

$T = T' + M_2g\sin\theta + f = T' + M_2g\sin\theta + \mu N$

$[\because f = \mu N = \mu M_2\, g\cos\theta]$

$\therefore T = T' + M_2g\sin\theta + \mu M_2g\cos\theta$ ...(ii)



Applying F_{net} = Ma for M_3 we get

$T' - f' = M_3 \times 0$

$\Rightarrow T' = f' = \mu N' = \mu M_3g$ ...(iii)

Putting the value of T and T′ from (i) and (iii) in (ii) we get

$M_1g = \mu M_3g + M_2g\sin\theta - \mu M_2g\cos\theta$

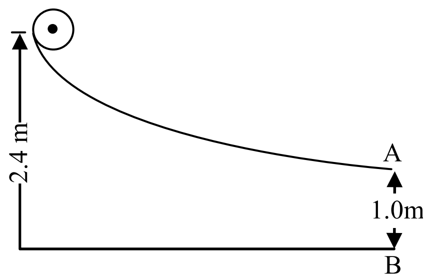
$M_1 = 0.25 \times 4 + 4 \times \sin 37º + 0.25 \times 4 \times \cos 37º$

$= 4.2$ kg

(b) The tension in the horizontal string will be

$T' = \mu M_3g = 0.25 \times 4 \times 9.8 = 9.8$ N

3. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track in a vertical plane. The track has an elevated section and a horizontal part, The horizontal part is 1.0 meter above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B(which is vertically below the end of the track as shown in fig.) where the sphere lands. During its flight as a projectile, does the sphere continue to rotate about its centers of mass ? Explain. **[IIT-1987]**



Sol. Applying law of conservation of energy at point D and point A

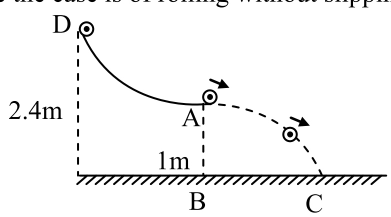
P.E. at D = P.E. at A + $(K.E.)_T$ + $(K.E.)_R$

$(K.E.)_T$ = Translational K.E.

$$mg\,(2.4) = mg\,(1) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$(K.E.)_R$ = Rotational K.E.

Since the case is of rolling without slipping



$\therefore$ v = rω

$\therefore \omega = \frac{v}{r}$ where r is the radius of the sphere Also

$I = \frac{2}{5}mr^2$

$$\therefore mg(2.4) = mg(1) + \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{v^2}{r^2}$$

$\Rightarrow$ v = 4.43 m/s

After point A, the body takes a parabolic path. The vertical motion parameters of parabolic motion will be

$u_y = 0$ $\qquad$ $S = ut + \frac{1}{2}at^2$

$S_y = 1m$ $\qquad$ $1 = 4.9\, t_y^2$

$a_y = 9.8\ m/s^2$

$\therefore t_y = ?$ $\qquad$ $t_y = \frac{1}{\sqrt{4.9}} = 0.45$ sec

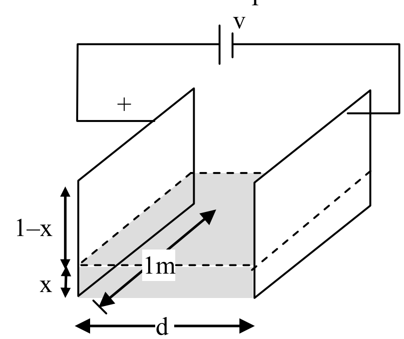
Applying this time in horizontal motion of parabolic path, BC = 4.43 × 0.45 = 2m

During his flight as projectile, the sphere continues to rotate because of conservation of angular momentum.

4. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of emf 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. Calculate the current drawn from the battery during the process. (Dielectric constant of oil = 11, $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-1}$)

[IIT-1994]

Sol. The adjacent figure is a case of parallel plate capacitor. The combined capacitance will be



$C = C_1 + C_2$

$$= \frac{k\varepsilon_0(x\times 1)}{d} + \frac{\varepsilon_0[(1-x)\times 1]}{d}$$

$$C = \frac{\varepsilon_0}{d}[kx + 1 - x]$$

After time dt, the dielectric rises by dx. The new equivalent capacitance will be

$C + dC = C_1' + C_2'$

$$= \frac{k\varepsilon_0}{d}[(x + dx) \times 1] + \frac{\varepsilon_0[1 - x - dx)\times 1]}{d}$$

dC = Change of capacitance in time dt

$$= \frac{\varepsilon_0}{d}[kx + kdx + 1 - x - dx - kx - 1 + x]$$

$$= \frac{\varepsilon_0}{d}(k-1)dx$$

$$\frac{dC}{dt} = \frac{\varepsilon_0}{d}(k-1)\frac{dx}{dt} = \frac{\varepsilon_0}{d}(k-1)v \qquad ...(i)$$

where $v = \frac{dx}{dt}$

We know that q = CV

$$\frac{dq}{dt} = V\frac{dC}{dt} \qquad ...(ii)$$

$$\Rightarrow \quad I = V\frac{\varepsilon_0}{d}(k-1)v$$

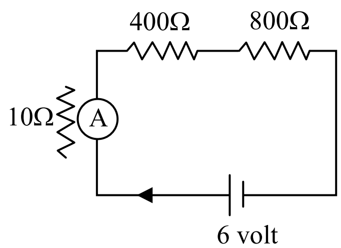
From (i) and (ii)

$$I = \frac{500 \times 8.85 \times 10^{-12}}{0.01}(11 - 1) \times 0.001$$

$= 4.425 \times 10^{-9}$ Amp.

5. Two resistors, 400 ohms, and 800 ohms are connected in series with a 6-volt battery. It is desired to measure the current in the circuit. An ammeter of a 10 ohms resistance is used for this purpose. What will be the reading in the ammeter? Similarly, If a voltmeter of 10,000 ohms resistance is used to measure the potential difference across the 400-ohms resistor, What will be the reading in the voltmeter. **[IIT-1982]**

Sol. Applying Kirchoff's law moving in clockwise direction starting from battery we get



$+ 6 - 10I - 400\,I - 800\,I = 0$

$\therefore\ 6 = 1210\,I$

$\therefore\ I = \frac{6}{1210} = 4.96 \times 10^{-3}$ A

The voltmeter and 400 Ω resistor are in parallel and hence p.d. will be same

$\therefore\ 10{,}000\,I_1 = 400\,I_2$ ...(i)

Applying Kircoff's law in loop ABCDEA starting from A in clockwise direction.

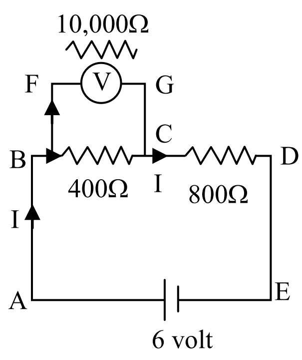
$- 400\,I_2 - 800\,I + 6 = 0$

$\therefore\ 6 = 400\,I_2 + 800\,(I_1 + I_2)$

$\therefore\ 6 = 400\,I_2 + 800(0.04\,I_2 + I_2)$

From (i) putting the value of I_1

$\therefore\ 6 = 1232\,I_2$



$\therefore\ I_2 = 4.87 \times 10^{-3}$ Amp.

Potential drop across 400 Ω resistor

$= I_2 \times 400$

$= 4.87 \times 10^{-3} \times 400$

$= 1.948$ volt ≈ 1.95 volt

$\therefore$ The reading measured by voltmeter = 1.95 volt

CHEMISTRY

6. A solution contains Na_2CO_3 and $NaHCO_3$.10 ml of this requires 2.0 ml of 0.1 M H_2SO_4 for neutralization using phenolphthalein as indicator. Methyl orange is then added when a further 2.5 ml of 0.2 M H_2SO_4 was required. Calculate the strength of Na_2CO_3 and $NaHCO_3$ in solution. **[IIT-1978]**

Sol. **Step 1**.

Equivalent mass of $Na_2CO_3 = \frac{\text{Molecular mass}}{2}$

$= \frac{106}{2} = 53$

Meq. of Na_2CO_3 in solution $= \frac{m_1}{53} \times 1000$

Step 2.

Equivalent mass of $NaHCO_3 = \frac{\text{Molecular mass}}{1}$

$= 84$

Meq. of $NaHCO_3$ in solution $= \frac{m_2}{84} \times 1000$

Step 3.

Meq. of H_2SO_4 used with phenolphthalein

= Valency factor × Molarity × Volume (ml)

$= 2 \times 0.1 \times 2.0 = 0.4$

$2Na_2CO_3 + H_2SO_4 \rightarrow 2NaHCO_3 + Na_2SO_4$

Meq. of H_2SO_4 used with phenolphthalein

$= \frac{1}{2}$ Meq. of Na_2CO_3 $\therefore$ $\frac{1}{2}$ Meq. of $Na_2CO_3 = 0.4$

Step 4.

Meq. of H_2SO_4 used with methyl orange

= Valency factor × molarity × volume(ml)

$= 2 \times 0.2 \times 2.5 = 1$

Meq. of H_2SO_4 used with methyl orange

= Meq. of $NaHCO_3$ + $\frac{1}{2}$ Meq. of Na_2CO_3

$\therefore$ Meq. of $NaHCO_3$ + $\frac{1}{2}$ Meq. of $Na_2CO_3 = 1$

$\therefore$ Meq. of $NaHCO_3 = 1 - 0.4 = 0.6$

and Meq. of $Na_2CO_3 = 2 \times 0.4 = 0.8$

Step 5.

$\frac{m_1}{53} \times 1000 = 0.8$ or $m_1 = \frac{0.8 \times 53}{1000} = 0.0424$

$\therefore$ Strength of Na_2CO_3 solution $= \frac{0.0424 \times 1000}{10}$

$= 4.24\ g\ L^{-1}$

Step 6.

$\frac{m_2}{84} \times 1000 = 0.6$ or $m_2 = \frac{0.6 \times 84}{1000} = 0.0504$

$\therefore$ Strength of $NaHCO_3$ solution $= \frac{0.0504 \times 1000}{10}$

$= 5.04\ g\ L^{-1}$

7. The molar volume of liquid benzene (density = 0.877 g ml^{-1}) increases by a factor of 2750 as it vaporizes at 20ºC and that of liquid toluene (density = 0.867 g ml^{-1}) increases by a factor of 7720 at 20ºC. A solution of benzene and toluene at 20ºC has a vapour pressure of 46.0 torr. Find the mole fraction of benzene in vapour above the solution. **[IIT-1996]**

Sol. Given that,

Density of benzene = 0.877 g ml^{-1}

Molecular mass of benzene (C_6H_6)

$= 6 \times 12 + 6 \times 1 = 78$

$\therefore$ Molar volume of benzene in liquid form $= \frac{78}{0.877}$ ml

$= \frac{78}{0.877} \times \frac{1}{1000}$ L = 244.58 L

And molar volume of benzene in vapour phse

$= \frac{78}{0.877} \times \frac{2750}{1000}$ L = 244.58 L

Density of toluene = 0.867 g ml^{-1}

Molecular mass of toluene ($C_6H_5CH_3$)

$= 6 \times 12 + 5 \times 1 + 1 \times 12 + 3 \times 1 = 92$

$\therefore$ Molar volume of toluene in liquid form

$= \frac{92}{0.867}$ ml $= \frac{92}{0.867} \times \frac{1}{1000}$ L

And molar volume of toluene in vapour phase

$= \frac{92}{0.867} \times \frac{7720}{1000}$ L = 819.19 L

Using the ideal gas equation,

$PV = nRT$

At $T = 20ºC = 293$ K

For benzene, $P = P_B^0 = \frac{nRT}{V}$

$= \frac{1 \times 0.082 \times 293}{244.58} = 0.098$ atm

= 74.48 torr ($\because$ 1 atm = 760 torr)

Similarly, for toluene,

$P = P_T^0 = \frac{nRT}{V}$

$= \frac{1 \times 0.082 \times 293}{819.19} = 0.029$ atm

= 22.04 torr ($\because$ 1 atm = 760 torr)

According to Raoult's law,

$P_B = P_B^0 x_B = 74.48\, x_B$

$P_T = P_T^0 x_T = 22.04\,(1 - x_B)$

And $P_M = P_B^0 x_B + P_T^0 x_T$

or $46.0 = 74.48\, x_B + 22.04\,(1 - x_B)$

Solving, $x_B = 0.457$

According to Dalton's law,

$P_B = P_M\, x'_B$ (in vapour phase)

or mole fraction of benzene in vapour form,

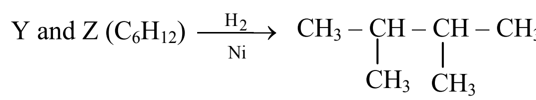
$x'_B = \frac{P_B}{P_M} = \frac{74.48 \times 0.457}{46.0} = 0.74$

8. An alkyl halide X, of formula $C_6H_{13}Cl$ on treatment with potassium t-butoxide gives two isomeric alkenes Y and Z(C_6H_{12}). Both alkenes on hydrogenation give 2, 3-dimethyl butane. Predict the structures of X, Y and Z. **[IIT-1996]**

Sol. The alkyl halide X, on dehydrohalogenation gives two isomeric alkenes.

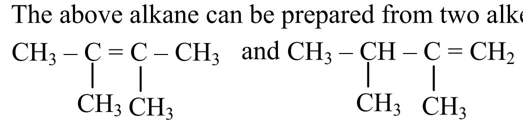
$$\underset{X}{C_6H_{13}Cl} \xrightarrow[\Delta;\,-HCl]{K-t-butoxide} \underset{C_6H_{12}}{Y + Z}$$

Both, Y and Z have the same molecular formula $C_6H_{12}(C_nH_{2n})$. Since, both Y and Z absorb one mol of H_2 to give same alkane 2, 3-dimethyl butane, hence they should have the skeleton of this alkane.



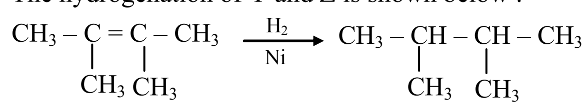
2,3-dimethyl butane

The above alkane can be prepared from two alkenes

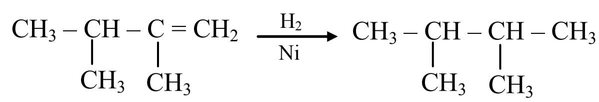


2,3-dimethyl butene-2 (Y)   2,3-dimethyl butene-1 (Z)

The hydrogenation of Y and Z is shown below :

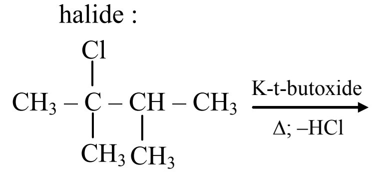


(Y)

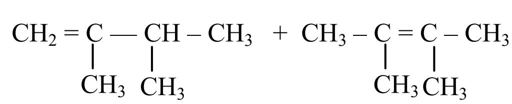


(Z)

Both, Y and Z can be obtained from following alkyl halide :



2-chloro-2,3-dimethyl butane (X)



(Z) 20%   (Y) 80%

Hence, X, $CH_3 - C(Cl)(CH_3) - CH(CH_3) - CH_3$

Y, $CH_3 - C(CH_3) = C(CH_3) - CH_3$

Z, $CH_3 - CH(CH_3) - C(CH_3) = CH_2$

9. A white precipitate was formed slowly when $AgNO_3$ was added to compound (A) with molecular formula $C_6H_{13}Cl$. Compound (A) on treatment with hot alcoholic KOH gave a mixture of two isomeric alkenes (B) and (C), having formula C_6H_{12}. The mixture of (B) and (C) on ozonolysis, furnished four compounds (i) CH_3CHO, (ii) C_2H_5CHO,

(iii) CH_3COCH_3 and

(iv) $CH_3 - CH(CH_3)–CHO$. What are the structures of (A) and (C) ? **[IIT-1986]**

Sol. It is given that,

$$\underset{\substack{(A)\\ \text{Alkyl chloride}}}{C_6H_{13}Cl} \xrightarrow[\text{KOH; –HCl}]{\text{Alcoholic}} \text{Two alkenes (B) + (C) with Formula } C_6H_{12}$$

$$\underset{\text{(B) and (C)}}{C_6H_{12}} \xrightarrow[\text{(ii) } H_2O/Zn]{\text{(i) } O_3} CH_3CHO + C_2H_5CHO + CH_3COCH_3 + CH_3 - CH(CH_3) - CHO$$

It is observed that during ozonolysis, no loss of carbon takes place, it may be concluded that CH_3CHO and $CH_3 - CH(CH_3) - CHO$ are the products of one alkene (B) and C_2H_5CHO and CH_3COCH_3 are the products of other alkene (say) (C). Thus, from the above we have :

$$CH_3 - C(H) = \boxed{O + O} = HC - CH(CH_3) - CH_3$$

$$\xrightarrow{-2[O]} \underset{(B)}{CH_3 - CH = CH - CH(CH_3) - CH_3}$$

$$\xrightarrow[\text{(ii) } H_2O/Zn]{\text{(i) } O_3} CH_3CHO + OHC - CH(CH_3) - CH_3$$

Similarly alkene (C) will be derived as :

$$(CH_3)_2C = \boxed{O + O} = CH.CH_2CH_3$$

$$\xrightarrow{-2[O]} \underset{(C)}{CH_3 - C(CH_3) = CH.CH_2CH_3}$$

$$\xrightarrow[\text{(ii) } H_2O/Zn]{\text{(i) } O_3} CH_3 - C(CH_3) = O + OHC.CH_2CH_3$$

Since the compounds (B) and (C) are obtained when (A), $C_6H_{13}Cl$, is dehydrohalogenated by heating it with alcoholic KOH, as follows :

$$CH_3CH_2 - CH(Cl) . CH(CH_3) - CH_3 \xrightarrow[\Delta;\ –HCl]{\text{Alc.KOH}}$$

$$\underset{\text{(B) (20\%) (minor)}}{CH_3 - CH = CH - CH(CH_3) - CH_3} + \underset{\text{(C) (80\%) (major)}}{CH_3CH_2CH = C(CH_3) - CH_3}$$

Since the Cl atom in (A) is an aliphatic chlorine, and it is attached to a secondary carbon atom which is adjacent to a tertiary cabon atom and one secondary carbon atom $\left[- CH_2 - CH(Cl) . CH(CH_3) - \right]$, it will react slowly with $AgNO_3$ to give a white precipitate.

Thus,

A, $CH_3 - CH_2 - CH(Cl) - CH(CH_3) - CH_3$

3-chloro-2-methyl pentane

B, $CH_3CH = CH - CH(CH_3) - CH_3$

4-methyl pentene -2

C, $CH_3CH_2CH = C(CH_3) - CH_3$

2-methyl pentene-2

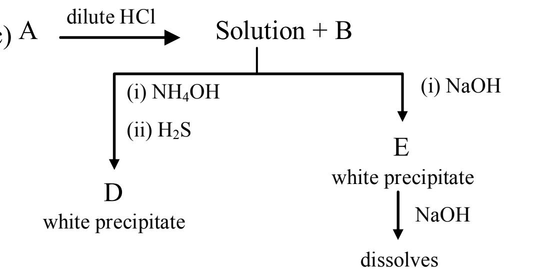
10. A white amorphous powder A when heated gives a colourless gas B, which turns lime water milky and the residue C which is yellow when hot but white when cold. The residue C dissolves in dilute HCl and the resulting solution gives a white precipitate on addition of potassium ferrocyanide solution. A dissolves in dilute HCl with the evolution of a gas which is identical in all respects with B. The solution of A as obtained above gives a white precipitate D on addition of excess of NH_4OH and on passing H_2S. Another portion of this solution gives initially a white precipitate E on addition of NaOH solution, which dissolves on further addition of the base. Identify the compound A to E. **[IIT-1979]**

Sol. The given information is as follows.

(a) $\underset{\text{white powder}}{A} \xrightarrow{\text{heat}} \underset{\substack{\text{colourless gas}\\\text{turns lime water}\\\text{milky}}}{B} + \underset{\substack{\text{residue}\\\text{yellow when}\\\text{hot white when cold}}}{C}$

(b) $C \xrightarrow{\text{dilute HCl}}$ solution $\xrightarrow{K_4Fe(CN)_6}$ white precipitate

(c)



From part (a), we conclude that B is CO_2 as it turns lime water milky :

$$Ca(OH_2) + CO_2 \rightarrow \underset{\substack{\text{milky due}\\\text{to this}}}{CaCO_3} + H_2O$$

and C is ZnO as it becomes yellow on heating and is white in cold. Hence, the salt A must be $ZnCO_3$.

From part (b), it is confirmed that C is a salt of zinc (II) which dissolves in dilute HCl and white precipitate obtained after adding $K_4[Fe(CN)_6$ is due to $Zn_2[Fe(CN)_6]$.

From part (c), it is again confirmed that A is $ZnCO_3$ as on adding dilute HCl, we get CO_2 and zinc (II) goes into solution. White precipitate is of ZnS which is precipitated in ammonical medium as its solubility product is not very low. White precipitate E is of $Zn(OH)_2$ which dissolves as zincate, in excess of NaOH. Hence the given information is explained as follows.

(a) $\underset{(A)}{ZnCO_3} \xrightarrow{\text{heat}} \underset{(B)}{CO_2} + \underset{(C)}{ZnO}$

(b) $\underset{(C)}{ZnO} \xrightarrow{\text{dil HCl}} \underset{\text{solution}}{ZnCl_2} \xrightarrow{K_4Fe(CN)_6} \underset{\text{White precipitate}}{Zn_2[Fe(CN)_6]}$

(c) $ZnCO_3 \xrightarrow{\text{dil HCl}} \underset{\text{Solution}}{ZnCl_2} + CO_2 + H_2O$

$$ZnCl_2 + S^{2-} \rightarrow \underset{(D)}{ZnS\downarrow} + 2Cl^-$$

$$Zn^{2+} + 2OH^- \rightarrow \underset{(E)}{Zn(OH)_2}$$

$$Zn(OH)_2 + 2OH^- \rightarrow \underset{\text{dissolves}}{ZnO_2^{2-}} + 2H_2O$$

MATHEMATICS

11. ABC is a triangle such that

$$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = \frac{1}{2}.$$

If A, B and C are in Arithmetic Progression, determine the values of A, B and C. **[IIT-1990]**

Sol. Given that in ΔABC, A, B and C are in A.P.

$A + C = 2B$

also $A + B + C = 180^o$

$\Rightarrow B = 60^o$

Also given that,

$\sin(2A + B) = \sin(C - A) = -\sin(B + 2C) = 1/2$ ...(1)

$\Rightarrow \sin(2A + 60^o) = \sin(C - A) = -\sin(60^o + 2C) = \frac{1}{2}$

$\Rightarrow 2A + 60^o = 30^o, 150^o$

{neglecting 30^o, as not possible}

$\Rightarrow 2A + 60^o = 150^o$

$\Rightarrow A = 45^o$

again from (1), $\sin(60^o + 2c) = -1/2$

$\Rightarrow 60^o + 2C = 210^o, 330^o$

$\Rightarrow C = 75^o$ or 135^o

Also from (1) $\sin(C - A) = 1/2$

$C - A = 30^o, 150^o, 195^o$

for $A = 45^o$, $C = 75^o$ and $C = 195^o$ (not possible)

$\therefore C = 75^o$

Hence, $A = 45^o, B = 60^o, C = 75^o$

12. Show that

$$\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$$

[IIT-1990]

Sol. Let, $I = \int_0^{\pi/2} f(\sin 2x)\sin x\,dx$ ...(1)

Then, $I = \int_0^{\pi/2} f\left\{\sin 2\left(\frac{\pi}{2} - x\right)\right\}.\sin\left(\frac{\pi}{2} - x\right)dx$

$= \int_0^{\pi/2} f\{\sin 2x\}\,.\cos x\,dx$ ...(2)

adding (1) and (2), we get

$2I = \int_0^{\pi/2} f(\sin 2x)\,.\,(\sin x + \cos x)\,dx$

$= 2\int_0^{\pi/4} f(\sin 2x)\,.\,(\sin x + \cos x)\,dx$

$= 2\sqrt{2}\int_0^{\pi/4} f(\sin 2x)\sin\left(x + \frac{\pi}{4}\right)dx$

$= 2\sqrt{2}\int_0^{\pi/4} f\left(\sin 2\left(\frac{\pi}{4} - x\right)\right).\sin\left(\frac{\pi}{4} - x + \frac{\pi}{4}\right)dx$

$$= 2\sqrt{2}\int_0^{\pi/4} f(\cos 2x)\,.\cos x\,dx$$

$$\therefore\ I = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\,.\cos x\,dx$$

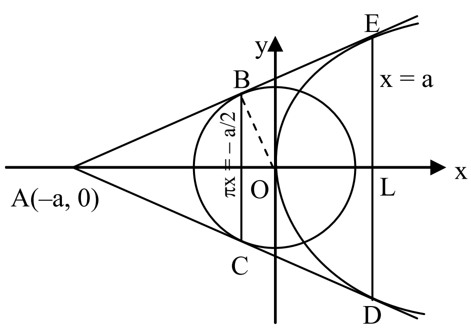
Hence, $\int_0^{\pi/2} f(\sin(2x))\,.\sin x\,dx$

$$= \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\,.\cos x\,dx$$

13. From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola. **[IIT-1996]**

Sol. Equation of any tangent to the parabola, $y^2 = 4ax$ is $y = mx + a/m$.

This line will touch the circle $x^2 + y^2 = a^2/2$



If $\left(\frac{a}{m}\right)^2 = \frac{a^2}{2}(m^2 + 1)$

$\Rightarrow\quad \frac{1}{m^2} = \frac{1}{2}(m^2 + 1) \Rightarrow\quad 2 = m^4 + m^2$

$\Rightarrow\quad m^4 + m^2 - 2 = 0$

$\Rightarrow\quad (m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow\quad m^2 - 1 = 0,\ m^2 = -2$ (which is not possible).

$\Rightarrow\quad m = \pm 1$

Therefore, two common tangents are

$y = x + a$ and $y = -x - a$

These two intersect at A$(-a, 0)$

The chord of contact of A$(-a, 0)$ for the circle $x^2 + y^2 = a^2/2$ is

$(-a)x + 0.y = a^2/2$ or $x = -a/2$

and chord of contact of A$(-a, 0)$ for the parabola $y^2 = 4ax$ is

$0.y = 2a(x - a)$ or $x = a$

Again length of BC = 2BK

$$= 2\sqrt{OB^2 - OK^2}$$

$$= 2\sqrt{\frac{a^2}{2} - \frac{a^2}{4}} = 2\sqrt{\frac{a^2}{4}} = a$$

and we know that DE is the latus rectum of the parabola so its length is 4a.

Thus area of the trapezium

BCDE = $\frac{1}{2}$(BC + DE) (KL)

$$= \frac{1}{2}(a + 4a)\left(\frac{3a}{2}\right) = \frac{15a^2}{4}$$

14. An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list on n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list ? **[IIT-2001]**

Sol. Let us define at onto function F from A : $[r_1, r_2 \ldots r_n]$ to B : [1, 2, 3] where $r_1r_2 \ldots r_n$ are the readings of n throws and 1, 2, 3 are the numbers that appear in the n throws.

Number of such functions,

$M = N - [n(1) - n(2) + n(3)]$

where N = total number of functions and

n(t) = number of function having exactly t elements in the range.

Now, $N = 3^n$, $n(1) = 3.2^n$, $n(2) = 3$, $n(3) = 0$

$\Rightarrow\quad M = 3^n - 3.2^n + 3$

Hence the total number of favourable cases

$= (3^n - 3.2^n + 3).\,{}^6C_3$

$\Rightarrow$ required probability $= \frac{(3^n - 3.2^n + 3)\times {}^6C_3}{6^n}$

15. Determine the equation of the curve passing through the origin in the from $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$ **[IIT-1996]**

Sol. $\frac{dy}{dx} = \sin(10x + 6y)$

Let $10x + 6y = t$ (given) .....(1)

$\Rightarrow\quad 10 + 6\frac{dy}{dx} = \left(\frac{dt}{dx}\right)$

$\Rightarrow\quad \frac{dy}{dx} = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$

Now the given differential equation becomes

$$\sin t = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$$

$\Rightarrow\quad 6\sin t = \frac{dt}{dx} - 10$

$\Rightarrow\quad \frac{dt}{dx} = 6\sin t + 10$

$\Rightarrow\quad \frac{dt}{6\sin t + 10} = dx$ apply variable separable

Integrating both the sides, we get

$$\int \frac{dt}{6\sin t + 10} = \int dx$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{dt}{3\sin t + 5} = x + c \qquad .....(2)$$

Let $\quad I_1 = \int \frac{dt}{3\sin t + 5}$

Put tan $t/2$ = u

$$\Rightarrow \quad \frac{1}{2}\sec^2 t/2\, dt = du$$

$$\Rightarrow \quad dt = \frac{2du}{\sec^2 t/2}$$

$$\Rightarrow \quad dt = \frac{2du}{1+\tan^2 t/2}$$

$$\Rightarrow \quad dt = \frac{2du}{1+u^2}$$

Also, $\quad I_1 = \int \frac{dt}{3\sin t + 5} = \int \frac{dt}{3\left(\frac{2\tan t/2}{1+\tan^2 t/2}\right)+5}$

$$= \int \frac{(1+\tan^2 t/2)dt}{(6\tan\frac{t}{2} + 5 + 5\tan^2\frac{t}{2})}$$

$$= \int \frac{2(1+u^2)du}{(1+u^2)(5u^2+6u+5)}$$

$$= \frac{2}{5}\int \frac{du}{u^2 + (6/5)u + 1}$$

$$= \frac{2}{5}\int \frac{du}{u^2 + \frac{6}{5}u + \frac{9}{25} - \frac{9}{25} + 1}$$

$$= \frac{2}{5}\int \frac{du}{\left(u+\frac{3}{5}\right)^2 + \frac{16}{25}}$$

$$= \frac{2}{5}\int \frac{du}{\left(u+\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \frac{2}{5}\cdot\frac{5}{4}\tan^{-1}\left(\frac{u+3/5}{4/5}\right)$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{5u+3}{4}\right]$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{5\tan t/2 + 3}{4}\right]$$

Putting this in (2)

Now $\quad \frac{1}{2} I_1 = x + c$

$$\Rightarrow \quad \frac{1}{4}\tan^{-1}\left[\frac{5\tan\frac{t}{2}+3}{4}\right] = x + c$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{5\tan\frac{t}{2}+3}{4}\right] = 4x + 4c$$

$$\Rightarrow \quad \frac{1}{4}[5\tan(5x+3y)+3] = \tan(4x+4c)$$

$\Rightarrow$ 5tan $(5x + 3y) + 3 = 4\tan(4x + 4c)$

When $x = 0$, $y = 0$ we get

$5\tan 0 + 3 = 4\tan(4c)$

$$\Rightarrow \quad \frac{3}{4} = \tan 4c$$

$$\Rightarrow \quad 4c = \tan^{-1}\frac{3}{4}$$

Then, $5\tan(5x + 3y) + 3 = 4\tan(4x + \tan^{-1} 3/4)$

$$\Rightarrow \quad \tan(5x+3y) = \frac{4}{5}\tan(4x+\tan^{-1}3/4) - \frac{3}{5}$$

$$\Rightarrow \quad 5x + 3y = \tan^{-1}\left[\frac{4}{5}\{\tan(4x+\tan^{-1}3/4\} - \frac{3}{5}\right]$$

$$\Rightarrow \quad 3y = \tan^{-1}\left[\frac{4}{5}\{\tan(4x+\tan^{-1}3/4\} - \frac{3}{5}\right] - 5x$$

$$\Rightarrow \quad y = \frac{1}{3}\tan^{-1}\left[\frac{4}{5}\{\tan(4x+\tan^{-1}3/4\} - \frac{3}{5}\right] - \frac{5x}{3}$$

Abundances of the Elements in the Earth's Crust

XTRA FUNDA

Element	Approximate % by weight
Oxygen	46.6
Silicon	27.7
Aluminum	8.1
Iron	5.0
Calcium	3.6
Sodium	2.8
Potassium	2.6
Magnesium	2.1
All others	1.5

Given the abundance of oxygen and silicon in the crust, it should not be surprising that the most abundant minerals in the earth's crust are the silicates. Although the Earth's material must have had the same composition as the Sun originally, the present composition of the Sun is quite different. The elemental composition of the human body and life in general is quite different.

These general element abundances are reflected in the composition of igneous rocks.

8 Physics Challenging Problems

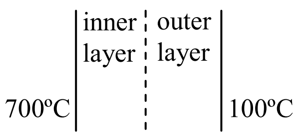
Set # 3

This section is designed to give IIT JEE aspirants a thorough grinding & exposure to variety of possible twists and turns of problems in physics that would be very helpful in facing IIT JEE. Each and every problem is well thought of in order to strengthen the concepts and we hope that this section would prove a rich resource for practicing challenging problems and enhancing the preparation level of IIT JEE aspirants.

Solutions will be published in next issue

By : Dev Sharma
Director Academics, Jodhpur Branch

1. The temperature drops through a two layer wall by 600ºC. Each layer is of equal of cross section. Which of the following actions will result in lowering the temperature θ of the interface



(A) By increasing the thermal conductivity of outer layer
(B) By increasing the thermal conductivity of inner layer
(C) By increasing thickness of outer layer
(D) By increasing thickness of inner layer

2. 'n' moles of an ideal monoatomic gas undergoes a process given by $T = KV^3$. If the temperature of the gas changes from T_0 to $3T_0$ then-
(A) work done by the gas is 2/3 nRT_0
(B) work done by the gas is 1/3 nRT_0
(C) molar specific heat for this process is 11/6 nRT_0
(D) Heat supplied is 11/6 nRT_0

3. An ideal gas having molar specific heat of constant volume C_V. If is undergoing a process where temperature is varying as $T = T_0 e^{\alpha V}$ where α is constant and "V' is the volume occupied by the gas. The molar specific heat of the gas for the given process as a function of volume is given by-

(A) $C_V + \frac{\alpha R}{V}$ (B) $C_V + \frac{R}{\alpha V}$

(C) $C_V + \frac{2\alpha R}{V}$ (D) $C_V + \frac{R}{2\alpha V}$

Passage # (Q. No. 4 to Q. No. 6)

Newton's law of cooling says for small temperature difference between a body and surrounding. The rate of cooling of the body is directly proportional to the temperature difference and surface area exposed. We can write $\frac{dT}{dt} = -bA(T - T_0)$.

The constant "b" depends on the nature of the surface involved and surrounding conditions. "T" is the temperature of object and T_0 is that of surrounding.

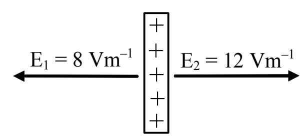
A metal block of heat capacity 100JºC placed in a room at 30ºC is heated electrically. The heater is switched off when the temperature of block reaches 50ºC. The temperature of block rises at the rate 2ºC per second just after the heater is switched on and falls at the rate 0.2ºC/sec just after heater is switched off. Assume Newton's law of cooling hold.

4. The power radiated by the block just after the heater is turned off –
(A) 10W (B) 20W (C) 40W (D) None

5. What will be the power radiated by the block when the temperature of block becomes 40ºC?
(A) 10W (B) 20W (C) 40W (D) 5W

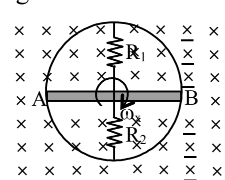
6. Assuming that the power radiated at 40ºC represents the average value of is the heating process. The time for which the heater was kept on-
(A) 5.2 sec (B) 10.5 sec (C) 21 sec (D) 15 sec

7. The electric field on two sides of a large charged plate is shown in figure. The charge density on the plate in S.I. units is given by (ε_0 is the permittivity of free space in S.I. units)



(A) $2\varepsilon_0$ (B) $4\varepsilon_0$ (C) $10\varepsilon_0$ (D) zero

8. AB is a resistanceless conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown. The resistance R_1 and R_2 do not rotate. Then current through the resistor R_1 is-



(A) $\frac{B\omega r^2}{2R_1}$ (B) $\frac{B\omega r^2}{2R_2}$

(C) $\frac{B\omega r^2}{2R_1R_2}(R_1 + R_2)$ (D) $\frac{B\omega r^2}{2(R_1 + R_2)}$

8 Solution
Physics Challenging Problems

Set # 2

Questions were Published in June Issue

1. During steady state $Q_{max} = 4C (C \to \mu F)$ at $t = t_1$ after charging,

$q = Q_{max}[1 - e^{-t_1/RC}]$

$q = 4C[1 - e^{-t_1/RC}]$ ....(i)

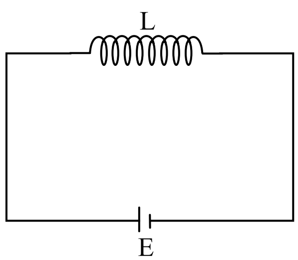
During discharging

$3 = (4C)e^{-t_1/RC}$ .....(ii)

Solving (i) and (ii) $\boxed{C = 3\mu F}$

Option [B] is correct

2. At any time $L\frac{di}{dt} = E$



$\int L di = \int E dt$

$Li = Et$

$i \propto t$

Option [B] is correct

3. It is possible only if after connecting voltmeter across 3Ω and 8Ω resistance it becomes balanced wheat stone Bridge.
So let resistance of voltmeter be $r\Omega$.

$\frac{2}{5} = \frac{R_1}{R_2}$ where $R_1 = \frac{3r}{r+3}, R_2 = \frac{8r}{r+8}$

Solving we get $r = 72\Omega$

Option [A] is correct

4. $A \to P, S, T; B \to Q, R; C \to P, S; D \to Q, R, T$
Hint: from lenz's law.

5. **Option [A] is correct**

6. Irms1 = Irms2

$$\Rightarrow \frac{1}{(\omega C)^2 + R^2} = \frac{1}{\left(\frac{1}{\omega C}\right)^2 + R^2}$$

$$\omega = \frac{1}{LC} \Rightarrow f = \frac{1}{2\pi}\sqrt{LC}$$

Option [D] is correct

7. Voltage is same for both in capacitor current leads voltage by $\cos^{-1}\frac{R}{\sqrt{X_C^2 + R^2}} = \phi$, in inductor current lags voltage by $\cos^{-1}\frac{R}{\sqrt{X_C^2 + R^2}} = \phi_2$.

$\phi_1 + \phi_2 = 90 \Rightarrow \cos(\phi_1 + \phi_2) = 0$

$\tan\phi_1 + \tan\phi_2 = 1$

$\frac{X_L}{R} \times \frac{X_C}{R} = 1 \Rightarrow L = CR^2$

Option [C] is correct

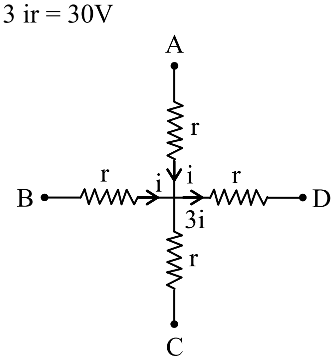
8. **[0030]**

V_{AD} = 40V (given)

Now, ir + 3ir = 40V

ir = 10V

V_{OD} = 3 ir = 30V



WHATS A LIGHT YEAR ?

A light year is the total distance travelled by a beam of light in one year. A beam of light can travel at up to 300,000 km/sec and the approximate distance covered in a year works out to be 9,470,000,000,000 km

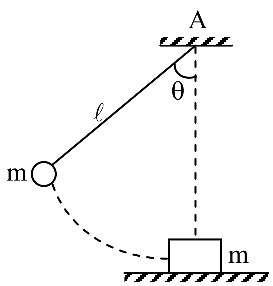
(1 light year = 9.47 x1012 km).

PHYSICS

Students' Forum

Expert's Solution for Question asked by IIT-JEE Aspirants

1. In the arrangement shown in figure ball and block have the same mass m = 1 kg each, $\theta = 60^o$ and length $\ell = 2.50$ m. Co-efficient of friction between block and floor is 0.5. When the ball is released from the position shown in the figure, it collides with the block and the block stops after moving a distance 2.50 m.



Find coefficient of restitution for collision between the ball and the block. ($g = 10\ ms^{-2}$)

Sol. When ball is released, it moves along a vertical circle with centre at A. Kinetic energy of ball just before collision is equal to loss of its potential energy from point of release to the point of collision. Therefore, velocity v_1 of ball, just before collision, is given by

$$\frac{1}{2} mv_1^2 = mg (\ell - \ell \cos \theta) \qquad \therefore\ v_1 = 5\ ms^{-1}.$$

After collision block starts to move towards right. But it is retarded by force of friction and ultimately it comes to rest.

According to law of conservation of energy,

kinetic energy of block just after collision = work done by it against friction.

Therefore, its velocity v_2, just after collision is given by $\frac{1}{2} mv_2^2 = \mu mgs$

where $\mu = 0.5$ and s = 2.50 m

$\therefore \quad v_2 = 5\ ms^{-1}$

co-efficient of restitution, $e = -\frac{v_2 - v_1}{u_2 - u_1}$

where $u_2 = 0$, $u_1 = 5\ ms^{-1}$ and $v_2 = 5\ ms^{-1}$

$\therefore \quad e = -\frac{5 - v_1}{0 - 5}$

or $\quad 5 - v_1 = 5e \quad$ or $v_1 = (5 - 5e) \qquad$ ...(i)

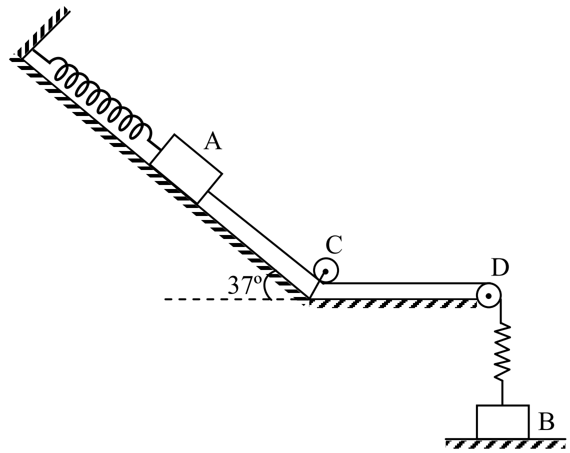
Applying law of conservation of momentum,

$mu_1 + nu_2 = mv_1 + mv_2$

or $5\ m + (m \times 0) = mv_1 + 5\ m \quad$ or $\quad v_1 = 0$

Substituting in equation (i) $\quad e = 1 \quad$ **Ans.**

2. A block A of mass m = 5 kg is attached with a spring having force constant $K = 2000\ Nm^{-1}$. The other end of the spring is fixed to a rough plane, inclined at 37º with horizontal and having coefficient of friction $\mu = 0.25$. Block A is gently placed on the plane such that the spring has no tension. Then block A is released slowly.



(i) Calculate elongation of the spring when equilibrium is achieved.

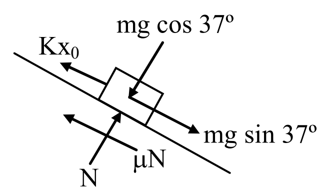
Now an inextensible thread is connected with block A and passed below pulley C and over pulley D, as shown in figure. Other end of the thread is connected with another block B of mass 3 kg. Block B is resting over a table and thread is loose.

If the table collapses suddenly and B falls freely through 80/9 cm the thread becomes taut, calculate

(ii) combined speed of blocks at that instant and

(iii) maximum elongation of spring in process of motion ($g = 10\ ms^{-2}$)

Sol. Since the block is released slowly, therefore it starts to slide down the plane till equilibrium of forces is achieved.



Let at that instant elongation of spring be x_0, then tension in it is $T_0 = Kx_0 = 2000x_0$.

Considering free body diagram of block, figure

$N = mg \cos 37^o = 40$ newton ...(i)

$Kx_0 + \mu N = mg \sin 37^o$ ...(ii)

From equation (i) and (ii)

$x_0 = 0.01$ m = 1 cm **Ans. (i)**

When table collapses, first block B falls freely under gravity through height 80/9 cm. Therefore, its speed just before the string becomes taut is

$$v_0 = \sqrt{2g \times \left(\frac{0.80}{9}\right)} = 4/3 \text{ ms}^{-1}.$$

Now block A is jerked into motion and a large tension (for a very small time interval) is developed in string due to which both the blocks A and B experience numerically equal impulses. Let its magnitude be J and let the combined speed of blocks be v,

Then for block A $J = 5v$ ...(iii)

For block B $3v_0 - J = 3v$ ...(iv)

From equation (iii) and (iv)

$v = 0.5 \text{ ms}^{-1}$ **Ans.(ii)**

At the instant of maximum elongation of spring, blocks are momentarily at rest.

Let distance moved by the blocks be x from the instant when block A was jerked into motion to the instant of maximum elongation of the spring.

According to law of conservation of energy,

loss of potential energy of A + loss of potential energy of B + loss of kinetic energy of blocks = increase in energy stored in spring + work done by the block A against friction

$$\therefore \; = 5.g(x \sin 37^o) + 3gx + \left\{\frac{1}{2}\times(5v^2) + \frac{1}{2}\times(3v^2)\right\}$$

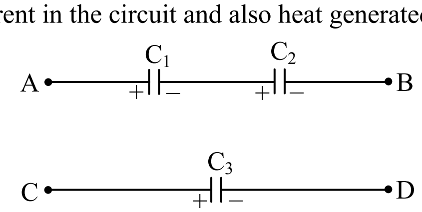
$$= \left\{\frac{1}{2}K(x_0 + x)^2 - \frac{1}{2}Kx_0^2\right\} + \mu Nx$$

$\therefore$ x = 0.05 m or 5 cm

$\therefore$ maximum elongation of spring = Its initial elongation (x_0) + Further elongation (x)

= 6 cm **Ans. (iii)**

3. Three capacitors $C_1 = 3\ \mu F$, $C_2 = 6\ \mu F$ and $C_3 = 6\ \mu F$ have equal charge q = 30 μC each. C_1 and C_2 are connected in series as shown in figure. If C_3 is connected across the series combination by connecting A with C and B with D and if resistance of connecting wires is R = 10 Ω, calculate initial current in the circuit and also heat generated.



Sol. Let initial current be I_0. Then at initial moment the circuit will be as shown in figure (i)

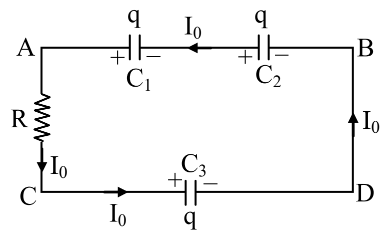


Fig. (i)

Applying Kirchhoff's voltage law on the circuit

$$\frac{q}{C_1} + \frac{q}{C_2} - \frac{q}{C_3} - I_0R = 0$$

$\therefore$ $I_0 = 1$ amp **Ans. (i)**

Heat generated in the circuit is equal to the loss of energy stored in capacitors.

initially energy stored, $U_1 = \frac{q^2}{2C_1} + \frac{q^2}{2C_2} + \frac{q^2}{2C_3}$

= 300 μJ

Let a charge Δq flows through the circuit till steady state is reached again.

Then charges on C_1, C_2 and C_3 become

$q_1 = (q - \Delta q)$, $q_2 = (q - \Delta q)$ and $q_3 = (q + \Delta q)$ respectively as shown in figure. (ii)

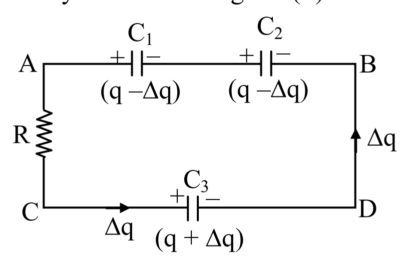


Fig. (ii)

Applying Kirchhoff's voltage law in final state

$$\frac{(q-\Delta q)}{C_1} + \frac{(q-\Delta q)}{C_2} - \frac{(q+\Delta q)}{C_3} = 0$$

$\therefore$ Δq = 15 μC

Therefore, finally energy stored in capacitors is

$$U_2 = \frac{(q-\Delta q)^2}{2C_1} + \frac{(q-\Delta q)^2}{2C_2} + \frac{(q+\Delta q)^2}{2C_3} = 225\ \mu J$$

$\therefore$ Heat generated = $U_1 - U_2$ = 75 μJ **Ans.**

4. A variable capacitor is adjusted in position of its lowest capacitance C_0 and is connected with a source of constant voltage V for a long time. Resistance of connecting wires is R. At t = 0, its capacitance starts to increase so that a constant current I starts to flow through the circuit. Calculate at time t

(i) power supplied by the source,

(ii) thermal power generated in the connecting wire and

(ii) rate of increase of electrostatic energy stored in capacitor.

(iv) What do you infer from above three results?

Sol. Since, Voltage V of the source is constant and circuit draws a constant current I from it, therefore, power supplied by the source is P = VI **Ans. (i)**

Thermal power generated in connecting wires,

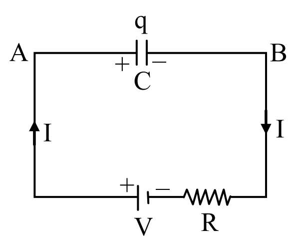
$$H = I^2R \qquad \textbf{Ans. (ii)}$$

Since, initial capacitance of the capacitor was equal to C_0 and it was connected with the source for long time, therefore, initial charge on capacitor was equal to

$q_0 = C_0V$

Since, a constant current I starts to flow at t = 0, therefore, at time t, charge on capacitor becomes equal to $q = (C_0V + It)$

At time t, circuit will be as shown in figure



Potential difference across the capacitor is

$V_C = V_A - V_B = (V - IR)$

$\therefore$ Electrostatic energy in capacitor at this instant is

$$U = \frac{1}{2} qV_C$$

Rate of increase of electrostatic energy = $\frac{dU}{dt}$

$$= \frac{1}{2}\frac{dq}{dt}V_C = \frac{1}{2}(V-IR)\,I$$

$$= \frac{1}{2}(VI - I^2R) \qquad \textbf{Ans. (iii)}$$

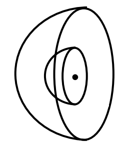
But power acting across the capacitor at this instant is $P_C = P - H = (VI - I^2R)$ while rate or increase of electrostatic energy in capacitor is half of it.

In fact, a force of attraction exists between surfaces of the capacitor. When these surfaces move towards each other capacitance increases. Hence, remaining part of the power acting across capacitor is used to increase kinetic energy of surfaces (plates) of the capacitor. **Ans. (iv)**

5. A Solid non-conducting hemisphere of radius R has a uniformly distributed positive charge of density ρ per unit volume. A negatively charged particle having charge q is transferred from centre of its base to infinity. Calculate work performed in the process.

Di-electric constant of material of hemisphere is unity

Sol. When negative charge q is displaced from centre of base to infinity, its electrical potential energy increases. Work is to be performed to increase this energy. To calculate initial potential energy of the particle, first a thin hemispherical shell of radius x and radial thickness dx is considered as shown in Figure



Volume of material of the shell = $2\pi x^2.dx$

$\therefore$ Charge on shell is $dQ = \rho(2\pi x^2\,dx)$

Since, every element of this shell is at a constant distance x from centre of curvature, therefore, potential energy of the particle, due to charge of the shell considered is

$$dU = \frac{1}{4\pi\varepsilon_0}\frac{(-q)(dQ)}{x} = -\frac{q\rho}{2\varepsilon_0}\,x\,dx$$

or total initial potential energy of particle,

$$U_0 = -\frac{q\rho}{2\varepsilon_0}\int_{x=0}^{x=R} x.dx = -\frac{q\rho R^2}{4\varepsilon_0}$$

When particle reaches infinity, its potential energy U becomes equal to zero.

$\therefore$ Work done = Increase in potential energy

$$= U - U_0 = \frac{q\rho R^2}{4\varepsilon_0} \qquad \textbf{Ans.}$$

Do you know

- 100 years ago: The first virus was found in both plants and animals.
- 90 years ago: The Grand Canyon became a national monument & Cellophane is invented.
- 80 years ago: The food mixer and the domestic refrigerator were invented.
- 70 years ago: The teletype and PVC (polyvinyl-chloride) were invented.
- 60 years ago: Otto Hahn discovered nuclear fission by splitting uranium, Teflon was invented.
- 50 years ago: Velcro was invented.
- 40 years ago: An all-female population of lizards was discovered in Armenia.
- 30 years ago: The computer mouse was invented.
- 20 years ago: First test-tube baby born in England, Pluto's moon, Charon, discovered.
- 10 years ago: First patent for a genetically-engineered mouse was issued to Harvard Medical School.

Capacitor-1

KEY CONCEPTS & PROBLEM SOLVING STRATEGY

Capacitance :

Whenever charge is given to a conductor of any shape its potential increases. The more the charge (Q) given to the conductor the more is its potential (V)

i.e. $Q \propto V$

$\Rightarrow$ $Q = CV$

where C is constant of proportionality called capacitance of the conductor C = Q/V, C = Q

SI unit of capacitance is farad (F) and 1 F = 1 coulomb/volt ($1CV^{-1}$)

Energy stored in a charged capacitor :

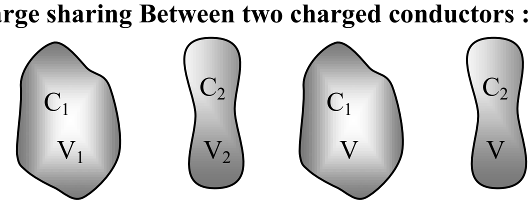
$$W = \frac{1}{2}CV_0^2 = \frac{Q^2}{2C} = \frac{1}{2}QV_0$$

Capacitance of an isolated sphere :

Let a conducting sphere of radius a acquire a potential V when a charge Q is given to it. The potential acquired by the sphere is

$$V = \frac{Q}{4\pi\varepsilon_0 a} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 a$$

Charge sharing Between two charged conductors :



$q_1 = C_1V_1$ $q_2 = C_2V_2$ $q'_1 = C_1V$ $q'_2 = C_2V$

(Initially) (Finally)

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

There is always a loss in energy during the sharing process as some energy gets converted to heat.

$$\text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

Capacitor or Condenser :

An arrangement which has capability of collecting (and storing) charge and whose capacitance can be varied is called **a capacitor (or condenser)**

The capacitance of a capacitor depends.

(a) directly on the size of the conductors of the capacitor.

(b) directly on the dielectric constant K of the medium between the conductors.

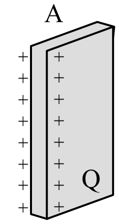
(c) inversely on the distance of separation between the conductor.

Principle of a condenser :

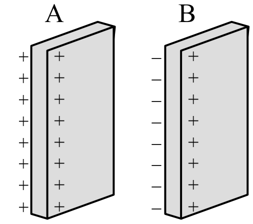
Consider a conducting plate A which is given a charge Q such that its potential rises to V. Then

$C = Q/V$

Let us place another identical conducting plate B parallel to it such that charge is induced on plate B (as shown in figure).



If V_- is the potential at A due to induced negative charge on B and V_+ is the potential at A due to induced positive charge on B, then



$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

Since $V' < V$ (as the induced negative charge lies closer to the plate A in comparison to induced positive charge).

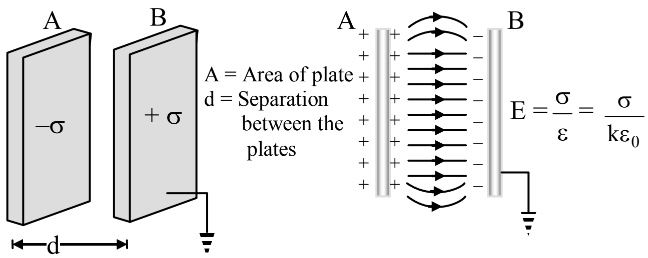
$\Rightarrow C' > C$

Further, if B is earthed from the outer side (see figure) then $V^n = V - V_-$ as the entire positive charge flows to the earth. So

$$C'' = \frac{Q}{V^n} = \frac{Q}{V - V_-} \Rightarrow C^n >> C$$

So, if an identical earthed conductor is placed in the viscinty of a charged conductor then the capacitance of the charged conductor increases appreciably. This is the principle of a parallel plate capacitor.

Parallel Plate Capacitor :



It consists of two metallic plates A and B each of area A at separation d. Plate A is positively charged and plate B is earthed. If K is the dielectric constant of the material medium and E is the field that exists between the two plates, then

$$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{K\varepsilon_0} \qquad \left\{\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right\}$$

$$\Rightarrow \frac{V}{d} = \frac{q}{K\varepsilon_0 A}$$

$$\Rightarrow C = \frac{q}{V} = \frac{K\varepsilon_0 A}{d}$$

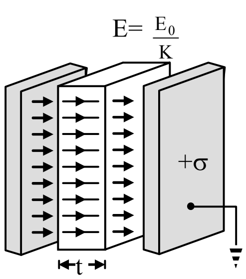
If medium between the plates is air or vacuum, then K = 1

$$\Rightarrow C_0 = \frac{\varepsilon_0 A}{d}$$

Special Case I :

When the space between the parallel plate capacitor is partly filled with a dielectric of thickness t(<d)

If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in a space d.



On inserting the slab of thickness t, a field $E = \frac{E_0}{K}$ exists inside the slab of thickness t and a field E_0 exists in remaining space (d – t). If V is total potential then

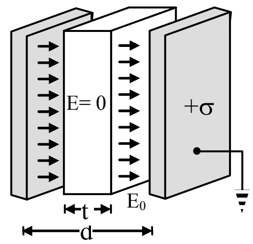
$$V = E_0(d - t) + Et$$

$$\Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Special Case II :

When the space between the parallel plate capacitor is partly filled by a conducting slab of thickness t(<d).

It no conducting slab is introduced between the plates, then a field $E_0 = \frac{\sigma}{\varepsilon_0}$ exists in a space d. If C_0 be the capacitance (without the introduction of conducting slab), then $C_0 = \frac{\varepsilon_0 A}{d}$



On inserting the slab, field inside it is zero and so a field $E_0 = \frac{\sigma}{\varepsilon_0}$ now exists in a space (d – t)

$$\Rightarrow V = E_0(d - t)$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}(d - t)$$

$$\Rightarrow V = \frac{q}{A\varepsilon_0}(d - t)$$

$$\Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t}$$

$$\Rightarrow C = \frac{\varepsilon_0 A}{d\left(1 - \frac{t}{dt}\right)}$$

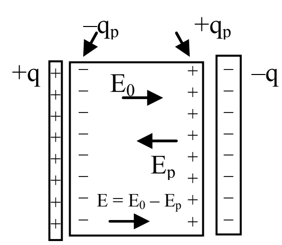
$$\Rightarrow C = \frac{C_0}{\left(1 - \frac{t}{d}\right)}$$

Since d – t < d

$\Rightarrow C > C_0$

i.e. Capacitance increases on insertion of conducting slab between the plates of capacitor.

Charge induced on a dielectric :



Resultant dielectric field within the plates is

$$E = E_0 - E_p$$

$$\Rightarrow E = \frac{1}{\varepsilon_0}(\sigma - \sigma_p) \quad ...(1)$$

$$\text{Also } E = \frac{\sigma}{K\varepsilon_0} \quad ...(2)$$

Compare (1) and (2), we get

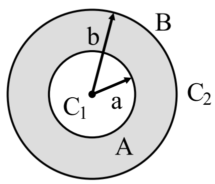
$$\frac{1}{\varepsilon_0}(\sigma - \sigma_p) = \frac{\sigma}{K\varepsilon_0}$$

$$\Rightarrow \sigma_p = \sigma\left(1 - \frac{1}{K}\right)$$

$$\Rightarrow \frac{q_p}{A} = \frac{q}{A}\left(1 - \frac{1}{K}\right)$$

$$\Rightarrow q_p = q\left(1 - \frac{1}{K}\right)$$

Spherical capacitor :



let C_1 be the capacitance in between the two conductors and C_2 be capacitance out side both.

To find C_1 :

Imagine the outer surface of B to be earthed. Then –q is the charge induced on the inner surface of B.

If V is the potential difference between the two surfaces, then

$$V = \frac{q}{4\pi\varepsilon_0 Ka} + \frac{-q}{4\pi\varepsilon_0 Kb}$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0 K}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\Rightarrow C = \frac{q}{V} = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right) \quad ...(1)$$

To find C,

Imagine A to be made open circuited (i.e. made non conducting), then

$$C_2 = 4\pi\varepsilon_0 Kb \quad ...(2)$$

Case I : When battery is connected to B and A is earthed. Then C_1 and C_2 are in parallel

$$\Rightarrow C = C_1 + C_2$$

$$\Rightarrow C = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right) + 4\pi\varepsilon_0 Kb$$

$$\Rightarrow C = 4\pi\varepsilon_0 K\left(\frac{b^2}{b-a}\right)$$

Case II : When battery is connected to A, then C_1 and C_2 are in series.

$$\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C} = \frac{b-a}{ab}\,\frac{1}{4\pi\varepsilon_0 K} + \frac{1}{4\pi\varepsilon_0 Kb}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{4\pi\varepsilon_0 Kb}\left(\frac{b-a}{a} + 1\right)$$

$$\Rightarrow \frac{1}{C} = \frac{1}{4\pi\varepsilon_0 Kb}\left(\frac{b}{a}\right)$$

$$\Rightarrow C = 4\pi\varepsilon_0 Ka$$

Case III : When battery connected to A and B is earthed. Then C_2 can be omitted as it will not receive any charge.

So, $C = C_1$

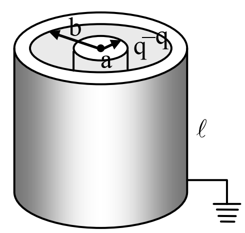
$$\Rightarrow \quad C = 4\pi\varepsilon_0 K\left(\frac{ab}{b-a}\right)$$

Case IV : When battery connected to B and A is open circuited (or made non conducted) then C_1 can be omitted (as it is open circuited). So,

$$C = C_2 \quad \Rightarrow \quad C = 4\pi\varepsilon_0 Kb$$

Cylindrical capacitor :

Let inner cylinder be given a charge per unit length of $\lambda\left(= \frac{q}{\ell}\right)$. A charge – q is induced on length ℓ at inner surface of outer cylinder



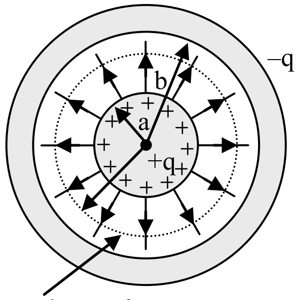
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ for } a < r < b$$

$$\Rightarrow -\frac{dV}{dr} = \frac{\lambda}{2\pi\varepsilon_0 Kr}$$

$$\Rightarrow \int_{\text{inner surface}}^{\text{outer surface}} dV = -\frac{\lambda}{2\pi\varepsilon_0 K}\int_{r=a}^{r=b}\frac{dr}{r}$$

$$\Rightarrow V_{\text{inner surface}} - V_{\text{outer surface}} = \frac{\lambda}{2\pi\varepsilon_0 K}\log_e\left(\frac{b}{a}\right)$$

Since, inner surface is at higher potential and outer at lower potential, so



$$\Rightarrow V_{\text{outer surface}} - V_{\text{inner surface}} = \frac{\lambda}{2\pi\varepsilon_0 K}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow V_{\text{inner surface}} - V_{\text{outer surface}} = \frac{q}{2\pi\varepsilon_0 \ell K}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow C = \frac{q}{V_{\text{inner surface}} - V_{\text{outer surface}}} = \frac{2\pi\varepsilon_0 \ell K}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow C = \frac{2\pi\varepsilon_0 \ell K}{\log_e\left(\frac{b}{a}\right)}$$

(A) Energy stored in a capacitor $E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$ and energy stored per unit volume $= \frac{1}{2}\varepsilon_0 E^2$

Note: The energy is stored in a capacitor is in the form of electric field between the plates.

(B) A parallel plate capacitor is charged by a battery and then the capacitor disconnected from the battery

(a) If the distance between plates of the capacitor is increased then the new parameter of the capacitors as compared to the previous parameters is

$q' = q$; $C' = \frac{\varepsilon_0 A}{d'}$, $V' = \frac{q'}{C'}$,

$E' = \frac{V'}{d'}$ (charge will not change) Energy $= \frac{1}{2}C'V'^2$

If a dielectric slab (dielectric constant k) is introduced between the plates then

$q' = q$, $C' = kC$, $V' = \frac{V}{k}$, $E' = \frac{E}{k}$ U'(Energy) $= \frac{U}{k}$

(charge will not change)

(C) A parallel plate capacitor is charged by a battery.

(a) If the distance between plates of the capacitor is Increased (with the battery connected) then the new parameters of the capacitors as compared to the previous parameters is

$V' = V$, $C' = \frac{\varepsilon_0 A}{d'}$, $q' = C'V'$, $E' = \frac{V'}{d}$,

Energy $U' = \frac{1}{2}C'V'^2$

(b) If a dielectric slab (dielectric constant k) is introduced between two plates then

$V' = V$, $C' = kC$, $q' = kq$, $E' = \frac{E}{k}$; $U' = KU$

(p.d.) will not change)

Solved Examples

1. A capacitor of 20 μF and charged to 500 volt is connected in parallel with another capacitor of 10 μF charged to 200 volt. Find the common potential.

Sol. Charge on one capacitor $q_1 = C_1V_1$

$\therefore$ $q_1 = 20 \times 10^{-6} \times 500 = 0.01$ coulomb

Charge on second capacitor

$q_2 = 10 \times 10^{-6} \times 200 = 0.002$ coulomb

The charge on the two capacitors

$q = q_1 + q_2 = 0.01 + 0.002 = 0.003$ coulomb

Total capacity $C = C_1 + C_2$

$= 20 \times 10^{-6} + 10 \times 10^{-6}$

$= 30 \times 10^{-6}$ Farad.

Common potential = q/C

$$= \frac{0.012}{30\times10^{-6}} = 400 \text{ Volt.}$$

2. A battery of 10V is connected to a capacitor of capacity of 0.1 F. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charge distributes equally on these two capacitors, find the total energy stored in the two capacitors. Further, compare this energy with the initial energy stored in the first capacitor.

Sol. The initial energy stored in the first capacitor.

$$U_0 = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 0.1 \times (10)^2 = 5.0 \text{ J}$$

When this capacitor is connected to the second uncharged capacitor, the charge distributes equally. This shows that the capacitance of the second capacitor is also C. The voltage across each capacitor will be V/2. If U be the energy stored in the two capacitors, then

$$U = \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2$$

$$= \frac{1}{4}CV^2 = 2.5 \text{ J}$$

$$\frac{U}{U_0} = \frac{2.5}{5.0} = \frac{1}{2}$$

3. Two isolated metallic solid spheres of radii R and 2R are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere.

Sol. Charge on smaller sphere

$Q_1 = 4\pi R^2 . \sigma$

Charge on bigger sphere

$Q_2 = 4\pi(2R)^2\sigma = 16\pi R^2\sigma$

$\therefore$ Total charge $Q = Q_1 + Q_2 = 20\pi R^2\sigma$ ...(1)

Capacitances of two spherical conductors are

$C_1 = 4\pi\varepsilon_0 R$ and $C_2 = 4\pi\varepsilon_0(2R)$

$\therefore$ Total capacitance

$C = C_1 + C_2 = 12\pi\varepsilon_0 R$ ...(2)

After connection, the common potential V is given by

$$V = \frac{Q}{C} = \frac{20\pi R^2\sigma}{12\pi\varepsilon_0 R} = \frac{5R\sigma}{3\varepsilon_0}$$

New charge on bigger sphere

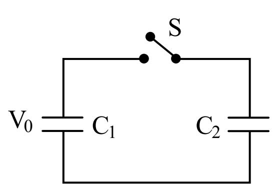
$Q_2' = C_2V$

$$= 4\pi\varepsilon_0 R(2R) \times (5R\sigma/3\varepsilon_0) = \frac{40\pi R^2\sigma}{3}$$

Surface density

$$\sigma_2' = \frac{Q_2'}{\text{surface area}} = \frac{\left(\frac{40\pi R^2\sigma}{3}\right)}{4\pi(2R)^2} = \frac{5}{6}\sigma.$$

4. A 8 μF capacitor C_1 is charged to V_0 = 120 volt. The charging battery is then removed and the capacitor is connected in parallel to an uncharged 4 μF capacitor C_2 **(a)** what is the potential difference V across the combination ? **(b)** What is the stored energy before and after the switch S is thrown ?



Sol. **(a)** Let q_0 be the charge on C_1 initially. Then

$q_0 = C_1 V_0$

when C_1 is connected to C_2 in parallel, the charge q_0 is distributed between C_1 and C_2. Let q_1 and q_2 be the charges on C_1 and C_2 respectively. Now let V be the potential difference across each condenser.

Now $q_0 = q_1 + q_2$

or $C_1V_0 = C_1V + C_2V$

$$\therefore \quad V = \frac{C_1}{C_1 + C_2}V_0 = \frac{8\mu F}{8\mu F + 4\mu F}(120\text{ V})$$

= 80 volt.

(b) Initial energy stored

$$U_0 = \frac{1}{2}C_1V_0^2$$

$$= \frac{1}{2}(8 \times 10^{-6})(120)^2$$

$= 5.76 \times 10^{-2}$ Joule

Final energy stored

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}(8 \times 10^{-6})(80)^2 + \frac{1}{2}(4 \times 10^{-6})(80)^2$$

$= 3.84 \times 10^{-2}$ joule

Final energy is less than the initial energy. The loss of energy appears as heat in connecting wires.

5. Calculate the capacitance of a parallel plate condenser, with plate area A and distance between plates d, when filled with a dielectric whose dielectric constant varies as

$$\varepsilon(x) = \varepsilon_0 + \beta x \qquad 0 < x < \frac{d}{2}$$

$$\varepsilon(x) = \varepsilon_0 + \beta(d - x) \qquad \frac{d}{2} < x < d$$

For what value of β would the capacity of the condenser be twice that when it is without any dielectric.

Sol. The capacitance in series is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad \frac{1}{C'} = \frac{1}{A}\times\left[\int_0^{d/2}\frac{dx}{\varepsilon_0 + \beta x} + \int_{d/2}^{d}\frac{dx}{\varepsilon_0 + \beta(d - x)}\right]$$

$$= \frac{1}{A\beta}[\{\log(\varepsilon_0 + \beta x)\}_0^{d/2} - \{\log(\varepsilon_0 + \beta(d - x)\}_{d/2}^{d}]$$

$$= \frac{1}{A\beta}\left[\left\{\log\left(\varepsilon_0 + \beta\frac{d}{2}\right) - \log\varepsilon_0\right\} - \left\{\log\varepsilon_0 - \log\left(\varepsilon_0 + \beta\frac{d}{2}\right)\right\}\right]$$

$$= \frac{2}{A\beta}\left[\log\left(\varepsilon_0 + \beta\frac{d}{2}\right) - \log\varepsilon_0\right]$$

$$= \frac{2}{A\beta}\log\left(\frac{\varepsilon_0 + \beta d/2}{\varepsilon_0}\right)$$

The capacitance C of a condenser without dielectric is given by

$$C = \frac{A\varepsilon_0}{d}$$

According to the question, $C' = 2C$

$$\therefore \quad \frac{2}{A\varepsilon_0}\log\left(\frac{\varepsilon_0 + \beta d/2}{\varepsilon_0}\right) = \frac{d}{2\varepsilon_0 A}$$

$$\beta = \frac{4\varepsilon_0}{d}\log\left(\frac{\varepsilon_0 + \beta d/2}{\varepsilon_0}\right)$$

Friction

KEY CONCEPTS & PROBLEM SOLVING STRATEGY

Friction :

Whenever there is a relative motion between two surfaces in contact with each other, an opposing force comes into play which forbids the relative motion of two bodies. This opposing force is called the force of friction.

Ex. : If a book on a table slides from left to right along the surface of a table, a frictional force directed from right to left acts on the book.

Frictional force may also exist between the surfaces when there is no relative motion. Frictional forces arise due to molecular interactions.

Static and Kinetic Friction :

The frictional force between two surface before the relative motion actually starts is called **static frictional force** or **static friction,** While the frictional force between two surfaces in contact and in relative motion is called kinetic frictional force or kinetic friction.

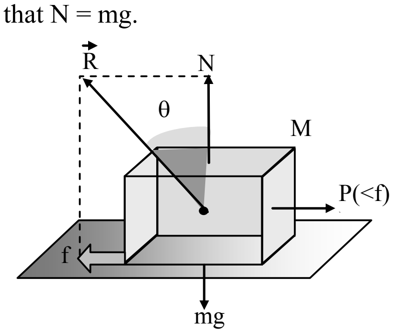
Static friction is a self adjusting force and it adjusts both in magnitude and direction automatically. Its magnitude is always equal to external effective applied force, tending to cause the relative motion and its direction is always opposite to that of external applied force.

So, when a body is not in motion or equilibrium, then

Force of static Friction = Applied External Force

Limiting friction, coefficients of friction and angle of friction :

Consider a block resting on a rough horizontal surface. The forces acting on the block are its weight mg downwards and normal reaction N acting upward. Such that N = mg.



Now suppose a force F_{app} is applied to the block to the right, then there will arise a frictional force f directed to the left (opposite to direction of applied force), which prevents the motion of the block. Let the resultant of $\vec{N}$ and $\vec{F}$ be $\vec{R}$ which makes an angle θ with normal reaction $\vec{N}$. Resolving $\vec{R}$ along $\vec{N}$ and $\vec{F}$, we get

$R \cos \theta = N$ and $R \sin \theta = f$

For equilibrium $N = mg$ and $f = F_{app}$

If we increase the pull F_{app} continuously, the force of friction increases and a stage comes when the body is just on the state of moving. This state is called limiting equilibrium. Under this condition the frictional force is maximum and is equal to applied force.

Limiting Friction :

The maximum value of static frictional force exerted between two surfaces in contact parallel to surfaces for a given normal force between when the body is on the verge of motion them is called limiting friction.

Angles of Friction :

Angle of friction (θ) is the angle which the resultant of force of static friction (f) and normal (N) makes with the normal reaction

The Coefficient of Friction (μ) :

It is defined as the ratio of limiting friction F to the normal reaction N between two surface in contact,

i.e., $\mu = F/N$ ...(3)

from figure, $\tan \theta = F/N$ ...(4)

Equation (3) and (4)

$\mu = \tan \theta$

Static and Kinetic Regions :

If a graph is plotted between applied force and frictional force, the graph is obtained. In figure AC is limiting or (maximum) static friction and BD is kinetic friction. Obviously, kinetic friction is less than static friction.

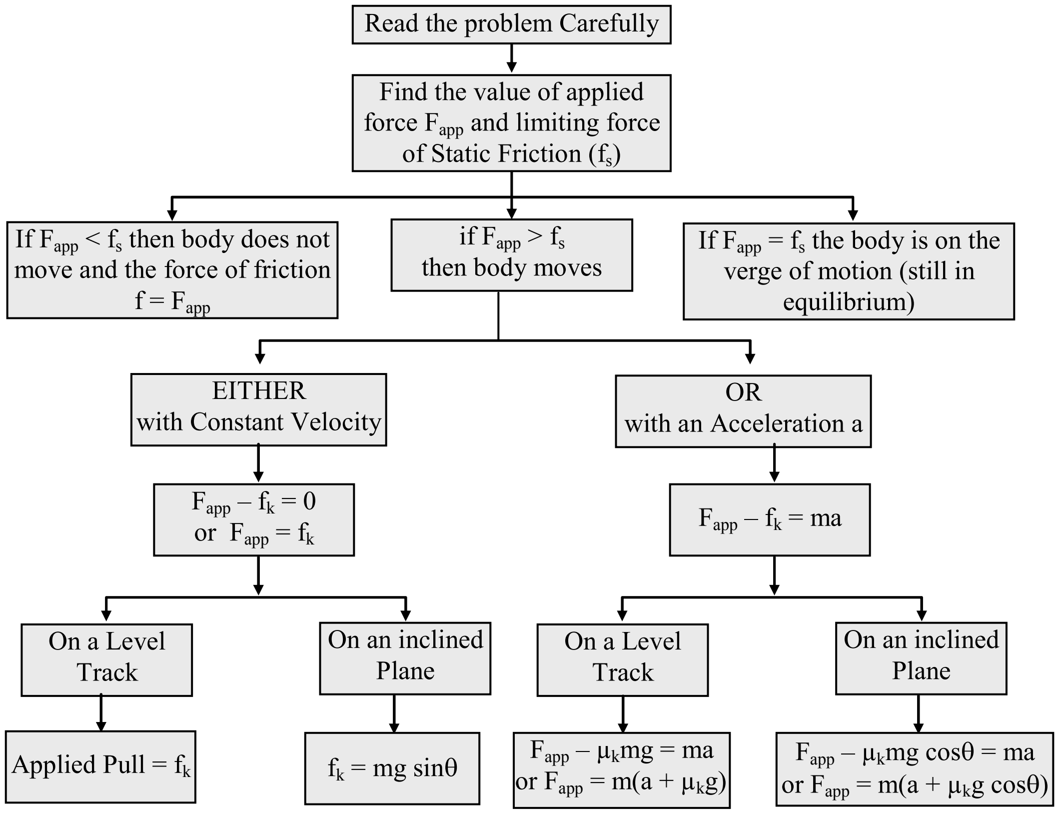
If relative motion is absent and is at the verge of start $\mu = \mu_s$, the coefficient of static friction but if relative motion is present $\mu = \mu_k$, the coefficient of kinetic friction.

The coefficient of friction depends on the

(a) strength of molecular interaction between the surfaces in contact,

(b) roughness of the two surface in contact.

Whenever we are dealing with problem involving friction we can follow the following analysis flow chart.



Laws of static and kinetic friction :

(a) The force of limiting friction is directly proportional to normal reaction for the same two surfaces in contact and acts opposite to direction of pull.

The kinetic friction is also proportional to normal reaction and acts opposite to direction of instantaneous relative motion. The kinetic friction is less than the static friction.

(b) The force of limiting (or static) friction is independent of area of contact of bodies as long as normal reaction remains the same.

The kinetic friction (to a good approximation) is independent of velocity, provided the velocity is neither too large nor too small.

Angle of repose (α)

This is concerned with an inclined plane on which a block rests, exerting its weight on the plane.

The angle of repose α is the angle which an inclined plane makes with the horizontal such that a body placed on it is on the verge of motion (is just about to loose the state of rest).

Under this condition the forces acting on the block are:

(a) its weight mg, downward,

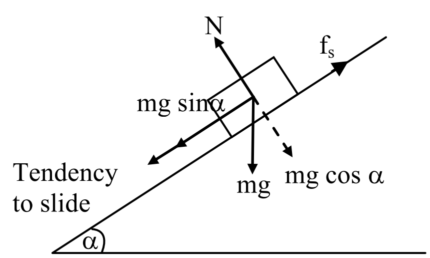
(b) normal reaction N, normal to plane,

(c) a force of friction f_s, parallel and tangential to plane upward.

Taking α as angle of inclination of the plane with the horizontal and resolving mg, parallel and perpendicular to inclined plane, then for equilibrium, we get

$N = mg \cos \alpha$ and $f_s = mg \sin \alpha$

$$\Rightarrow \tan \alpha = \frac{f_s}{N}$$



Frictional force on a bicycle in motion :

(a) When a wheel is rotated about its axle without sliding, the frictional force acting on it is the rolling friction and it acts opposite to the direction of tendency of motion of a points of its contacts with the ground. In case the wheel rotates clockwise and frictional force (f) on wheel is forward. In case the wheel rotates anticlockwise, the frictional force (f) on wheel is backward.

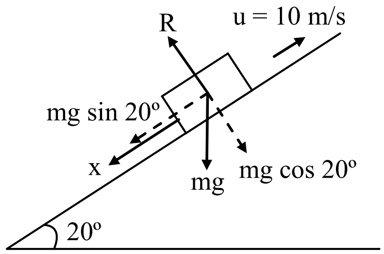
(b) When the bicycle is pedalled, the force exerted on the rear wheel through the pedal-chain-axle system is in backward direction, therefore force of friction on rear wheel is forward. The front wheel of cycle moves by itself in forward direction, hence the force of friction of front-wheel is in backward direction.

(c) When the bicycle is not pedalled, no external force is being exerted, both wheels move forward by itself due to inertia and so the net frictional force on both wheels is in backward direction.

Solved Examples

1. A block of mass 5 kg is placed on a slope which makes an angle of 20º with the horizontal and is given a velocity of 10 m/sec up the slope. Assuming that the coefficient of sliding friction between the block and the slope is 0.20, find how far the block travels up the slope ? Take g = 10 m/sec^2.

Sol. This situation is shown in fig.



The component of the weight perpendicular to plane
= mg cos 20º = 5 × 10 × 0.9397 = 46.98 N
The component of the weight parallel to the plane
= mg sin 20º = 5 × 10 × 0.3420 = 17.10 N
From figure R = mg cos 20º = 46.98 N
Here the coefficient of kinetic friction = 0.2
Thus the frictional force X = 0.2 × 46.98 = 9.39 N
The frictional force will be downward because the motion is in the upward direction.
The resultant force parallel to the plane is given by
= X + mg sin 20º = 9.39 + 17.10 = 26.49 N
From Newton's law F = ma, i.e., 26.49 = 5 × a

$\therefore\ a = \frac{26.49}{5} = 5.29$ m/s^2 downward

When the block is given a velocity 10 m/s in the upward direction we have
u = 10 m/s, v = 0, a = – 5.9 m/s^2.
(Taking the direction up the plane as positive)
Let s be the distance traveled by the block.
Using the formula $v^2 = u^2 + 2as$, we have

$0 = (10)^2 - 2 \times 5.29 \times s$

or $s = \frac{100}{2 \times 5.29} = 9.45$ m.

2. A block is projected up with 10 m/s along a fixed inclined plane of inclination 37º with the horizontal. If the time of ascend from the point of projection is half the time of descend to the same point, find the distance travelled by the block during the up and down journey.

Sol. Let μ, t_1 and t_2 be the coefficient of friction between the plane and the block, time of ascend and time of descend respectively.
The retardation while going up

$$a_1 = g(\sin\theta + \mu\cos\theta) = 10\left(\frac{3}{5} + \mu\frac{4}{5}\right)$$

The acceleration while descending

$$a_2 = g(\sin\theta - \mu\cos\theta) = 10\left(\frac{3}{5} - \mu\frac{4}{5}\right)$$

Now, s = distance of ascend = distance of descend.
As final velocity is zero, we have

$0 = u - a_1t_1$ or $u = a_1t_1$

Now $s = a_1t_1^2 - \frac{1}{2}a_1t_1^2 = \frac{1}{2}a_1t_1^2$

$s = a_1t_1^2 = \frac{1}{2}a_2t_2^2$ and $t_2 = 2t_1$

$$\therefore\ \left(\frac{a^2}{a_1}\right) = \left(\frac{t_1}{t_2}\right)^2 \quad\text{or}\quad \frac{\left(\frac{3}{5} - \mu\frac{4}{5}\right)}{\left(\frac{3}{5} + \mu\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^2$$

Solving we get μ = (9/20)

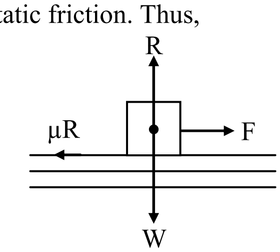
Again $a_1 = 10\left(\frac{3}{5} + \frac{9}{20} \times \frac{4}{5}\right) = 9.6$ m/sec^2

$\therefore\ s = \frac{u^2}{2a_1} = \frac{(10)^2}{2 \times 9.6} = 5.21$ meter

So total distance = 2s = 10.42 metre

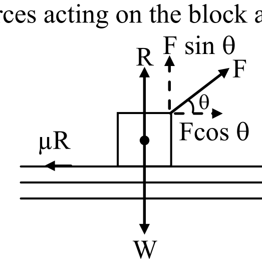
3. A block weighing 20 nt is at rest on a horizontal table. The coefficient of static friction between block and table is 0.50. **(a)** What is the magnitude of the horizontal force that will just start the block moving ? **(b)** What is the magnitude of a force acting upward 60º from the horizontal that will just start the block moving ? **(c)** If the force acts down at 60º from the horizontal how large can it be without causing the block to move ?

Sol. **(a)** As shown in fig. the horizontal force F that will just start the block moving is equal to the maximum force of static friction. Thus,



F = μR = μW = 0.50 × 20 nt. = 10.0 nt.

(b) The forces acting on the block are shown in fig.



The applied force is inclined at an angle θ in the upward direction. Its horizontal and vertical components are F cos θ and F sin θ respectively. In equilibrium.

$F\cos\theta = \mu R$ and $F\sin\theta + R = W$

or $R = (W - F\sin\theta)$

$\therefore$ $F\cos\theta = \mu(W - F\sin\theta) = \mu W - \mu F\sin\theta$

$F(\cos\theta + \mu\sin\theta) = \mu W$

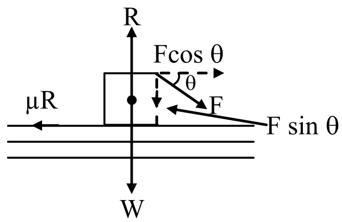
or $F = \dfrac{\mu W}{\cos\theta + \mu\sin\theta}$

Here μ = 0.50, W = 20 nt. and θ = 60º

$$F = \frac{0.50\times 20}{\cos 60^\circ + 0.5\sin 60^\circ} = \frac{10}{0.50 + 0.5\times 0.866}$$

= 10.72 nt.

(c) In this case,



$F\cos\theta = \mu R$ and $R = W + F\sin\theta$

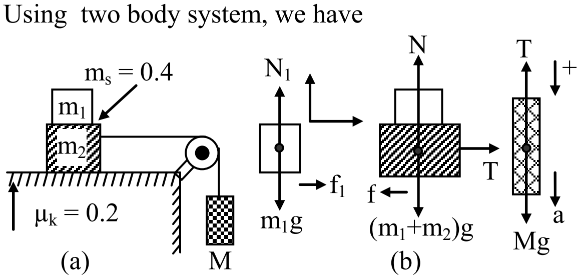
Solving we get,

$$F = \frac{\mu W}{\cos\theta - \mu\sin\theta} = \frac{0.50\times 20}{0.50 - 0.5\times 0.866}$$

= 149.2 nt.

4. Two blocks, m_1 = 2kg and m_2 = 4kg, are connected with a light string that runs over a frictionless peg to a hanging block with a mass M as shown in fig. (a). The coefficient of sliding friction between block m_2 and the horizontal surface at the speeds involved is $\mu_k = 0.2$. The coefficient of static friction between the two blocks is $\mu_S = 0.4$. What is the maximum mass M for the hanging block if the block m_1 is not to slip on block m_2 while m_2 is sliding over the surface ?

Sol. The relevant free body diagrams are shown in fig.(b) Using two body system, we have



$N - (m_1 + m_2)g = 0$ ...(1)

$T - F = (m_1 + m_2)a$ ...(2)

For hanging block

$Mg - T = Ma$ ...(3)

From eqs. (2) and (3),

$Mg - f = (M + m_1 + m_2)a$

But $f = \mu_k N = \mu_k(m_1 + m_2)g$ [$\because$ using eq. (1)]

$\therefore$ $Mg - \mu_k(m_1 + m_2)g = (m + m_1 + m_2)a$

or $a = \dfrac{\{M - \mu_k(m_1 + m_2)\}g}{(M + m_1 + m_2)}$ ...(4)

From free body diagram of mass m_1, we have

$N_1 - m_1 g = 0$ and $f_1 = m_1 a$

It should be noticed that the force f_1 accelerates m_1 to the right. Just before slipping occurs, we find

$$\frac{f_1}{N_1} = \mu_S \text{ or } \mu_S = \frac{m_1 a}{m_1 g} = \frac{a}{g}$$

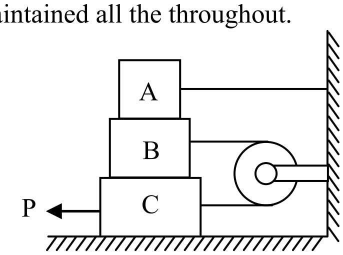
$\therefore$ $\mu_S = \dfrac{\{M - \mu_k(m_1 + m_2)\}}{(M + m_1 + m_2)}$ ....(5)

Solving eq. (5) for M, we have

$$M = \frac{(\mu_S + \mu_k(m_1 + m_2)}{1 - \mu_S}$$

or $M = \dfrac{(0.4 + 0.2)(2kg + 4kg)}{(1 - 0.4)}$ = 6 kg.

5. In fig.(a) the blocks A, B and C weight are 3kg, 4kg and 8kg respectively. The coefficient of sliding friction between any two surfaces is 0.25. A is held at rest by a massless rigid rod fixed to the wall, while B and C are connected by a light flexible cord passing around a fixed frictionless pulley. Find the force P necessary to drag C along the horizontal surface to the left at a constant speed. Assume that the arrangement shown in the diagram, B on C and A on B is maintained all the throughout.



Sol. When block C moves towards left, B moves towards right, while A is fixed. There would be a tension T in the string. Under this condition, let us consider the frictional forces between different surfaces.

Frictional force between A and B

= μR = 0.25 × 3

Frictional force between C and B

= μR = 0.25 (3 + 4) = 0.25 × 7

Frictional force between C and surface

= 0.25(3 + 4 + 8) = 0.25 × 15

Considering fig. (b)

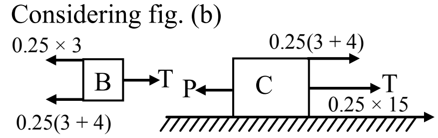


Fig (b)

Tension in the string = Frictional forces at upper and lower surfaces of block B

or T = 0.25 × 3 + 0.25 × 7 = 2.5 kg wt.

For block C,

P = T + Frictional force between C and B + Frictional force between C and surface

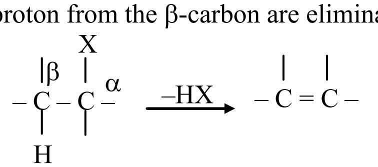
= 2.5 + 0.25 × (3 + 4) + 0.25 × (15) = 8 kg wt.

= 8 × 9.8 = 78.4 newton

REACTION MECHANISM

Elimination reactions :

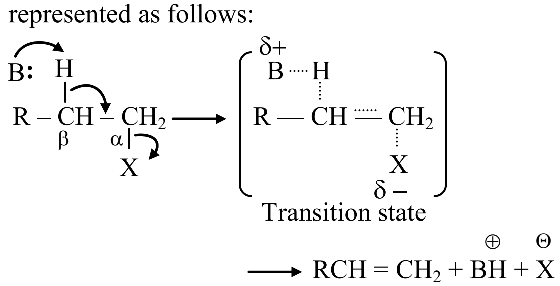
The elimination reactions are reverse of addition reactions. In these reactions two atoms or group attached to the adjacent carbon atoms of the substrate molecule are eliminated to form a multiple bond. In these reactions a atom or group from α-carbon atom and a proton from the β-carbon are eliminated.



In eliminations reactions, the presence of one hydrogen on the β-carbon atom is necessary. In general the elimination reactions are divided into two types, i.e., bimolecular elimination reactions (E_2) and unimolecular elimination reactions (E_1).

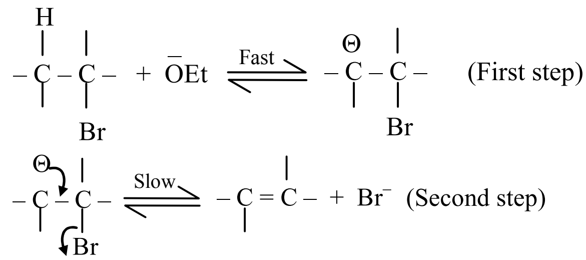
Bimolecular elimination reactions (E_2) :

In these elimination reactions, the rate of elimination depends on the concentration of the substrate and the nucleophile and the reaction is of second order. It is represented as E2. Like S_N2 reaction, the E2 reaction is also one step process. In these reactions abstraction of proton from the β-carbon atom and the expulsion of an atom or group from the α-carbon atom occur simultaneously. The mechanism of this reaction is represented as follows:



The above reaction is a one step process and passes through a transition state. This reaction is also known as 1, 2-elimination or simply β-elimination. In these reactions, the two groups to be eliminated (i.e., H and X) are trans to each other and hence E2 reactions are generally trans elimination.

The second-order elimination reaction may also proceed in two steps (as in E1 elimination which will be discussed subsequently). In this mechanism, the base removes the hydrogen in the first step to form an intermediate carbanion. In the second step, the intermediate carbanion looses the leaving group. The second step is slow and is rate determining step.



The rate of this reaction is dependent on the carbanion (conjugate base of the substrate). So this mechanism is called **ElcB mechanism** (Elimination, Unimolecular from conjugate base).

E1cB mechanism is not common for the E2 reactions. The carbanion mechanism occurs only where the carbanion from the substrate is stabilized and where the leaving group is a poor leaving group. A typical example, which follows E1cB mechanism is the formation of 1,1-dichloro-2,2-difluoroethene from 1,1-dichloro-2,2,2-trifluoroethane in presence of sodium ethoxide.

$$\underset{\text{1,1-Dichloro-2,2,2-trifluoroethane}}{CHCl_2-CF_3} \xrightleftharpoons{C_2H_5ONa} \underset{\text{Carbanion}}{Cl_2\bar{C}-CF_3} \xrightarrow{-F^-} \underset{\text{1,1-Dichloro-2,2-difluoroethane}}{Cl_2C=CF_2}$$

In the above case the carbanion is strongly stabilized due to –I effect of halogens. Also F^- is a poor leaving group.

A distinction between the E2 and E1cB mechanism can be made by tracer experiments. Thus, the reaction of 1-bromo-2-phenylethane (this substrate was selected as Ph group is expected to increase the acidity of β-hydrogen and also to stabilize the carbanion) with C_2H_5OD gives back the starting 1-bromo-2-phenylethane. If the carbanion mechanism had operated, the deuterium would have been found in the recovered 1-bromo-2-phenylethane, which is not the case.

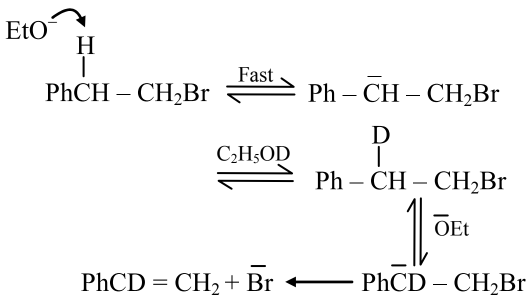
$$\underset{\text{1-Bromo-2-phenylethane}}{C_6H_5CH_2CH_2Br} + C_2H_5O^-$$

$$\rightleftharpoons C_6H_5\bar{C}HCH_2Br + C_2H_5OH$$

$$C_6H_5\bar{C}HCH_2Br + C_2H_5OD$$

$$\rightleftharpoons C_6H_5CH(D)CH_2Br + \bar{O}C_2H_5$$

In case the above reaction is allowed to go to completion, the product obtained will be



Styrene

The styrene obtained does not contain any deuterium (contrary to what has been shown in the above E1cB mechanism). So in the above reaction E2 mechanism operates.

The E2 mechanism is supported by the following evidences.

(i) During elimination, there is no rearranged product obtained. This is due to the fact that E2 is a single step process and does not involve the formation of intermediate carbocation (the carbocations are known to undergo rearrangement).

(ii) The E2 mechanism finds support from isotope labeling experiments. Dehydrohalogenation of unlabelled 1-bromopropane is seven times faster than the dehydrohalogenation of $CH_3CD_2CH_2Br$.

$$CH_3CH_2CH_2Br \xrightarrow{E2} CH_3CH=CH_2$$

$$CH_3CD_2-CH_2Br \xrightarrow{E2} CH_3CD=CH_2$$

In E2 mechanism a hydrogen (from $CH_3CH_2CH_2Br$) or a deuterium (from $CH_3CD_2CH_2Br$) has to be abstracted. It is known that the C – D bond is stronger than the C – H bond and requires more energy to be broken. Therefore, rate of elimination in $CH_3CD_2CH_2Br$ should be slower. In fact, it has been found that in the unlabelled alkyl halides the elimination rate is seven times more than in labelled alkyl halides.

Unsymmetrical substrate which has hydrogen attached to two different β-carbons can affored two alkenes. For example, 2-bromobutane on dehydrohalogenation may give 1-butene or 2-butene.

$$CH_3-CH(Br)CH_2CH_3 \xrightarrow{-HBr} CH_2=CHCH_2CH_3 + CH_3CH=CHCH_3$$

2-Bromobutane → 1-Butene + 2-Butene

In a similar way, decomposition of sec-butyl-trimethylammonium hydroxide may give a mixture of two alkenes. The question arises as to which alkene will be obtained in major amount in the above dehydrohalogenation. The orientation of the reaction is determined by **Hafmann** and **Saytzeff Rule**.

Hofmann Rule : This rule is applicable for those substrates in which α-carbon atom is attached to a positively charged atom. According to this rule, in the elimination reaction of positively charged species, the major product will be the alkene which is least substituted.

$$CH_3CH_2\overset{+}{N}(CH_3)_2CH_2CH_2CH_3\ \ \overline{O}H \xrightarrow{Heating} CH_2=CH_2 + CH_3CH_2CH_2N(CH_3)_2$$

$$CH_3CH_2\overset{+}{S}(CH_3)_2 \xrightarrow{C_2H_5\overline{O}} CH_2=CH_2 + S(CH_3)_2$$

Saytzeff Rule : In case of unsymmetrical alkyl halides, for example in 2-bromobutane, the course of elimination is determined by Saytzeff Rule. According to this rule, hydrogen is eliminated preferentially from the carbon atom which has less number of hydrogen atoms and so the highly substituted alkene is the major product.

$$CH_3CH_2-CH(Br)-CH_3 \xrightarrow{alk.KOH} CH_3CH=CHCH_3 + CH_3CH_2CH=CH_2$$

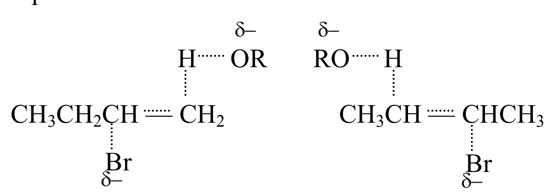
2-Bromobutane → 2-Butene(80%) + 1-Butene (20%)

$$CH_3CH_2-C(CH_3)(Br)-CH_3 \xrightarrow{C_2H_5O^-} CH_3CH=C(CH_3)-CH_3 + CH_3CH_2C(CH_3)=CH_2$$

2-Bromo-2-methylbutane → 2-methyl-2-butene (71%) + 2-methyl-1-butene (29%)

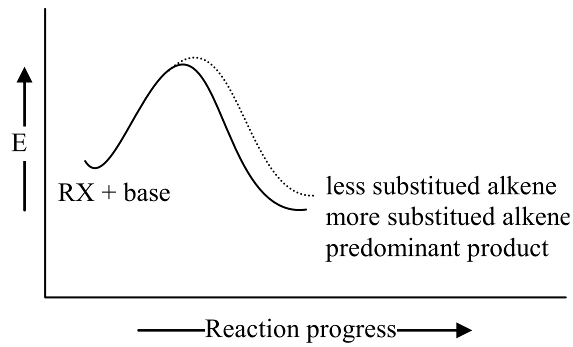
The formation of highly substituted alkene can be explained as follows.

The transition states of less substituted and more substituted alkenes from an alkyl halide are represented as shown below:



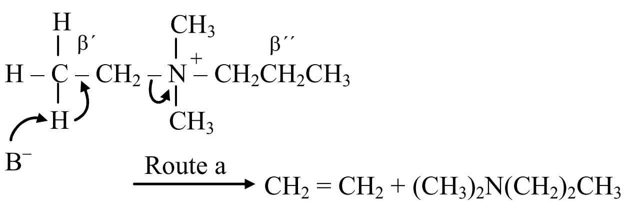
T.S. of less substituted alkene T.S. of more substituted alkene

Both the transition states have partial double bond character. However, the transition state leading to more stable alkene is more stabilized and is of lower energy. Thus, the more stable alkene is formed as the major product.

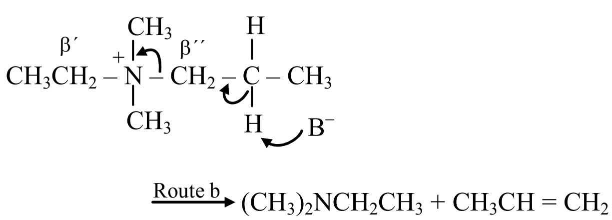


Energy diagram for a typical E2 reaction, showing why the more substituted alkene predominates

Hofmann rule can be understood by considering the mechanism of elimination reaction of quaternary ammonium hydroxide.

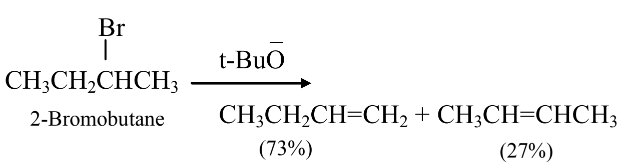


Another possibility is :



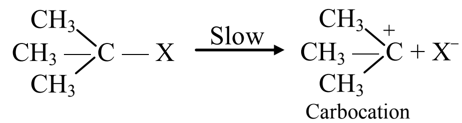
In the above reaction the strong electron-withdrawing group makes the hydrogens of the β-carbons more acidic for facile abstraction by the base. In this compound, with alternate β-hydrogens (marked β′ and β′′), the β" hydrogen are less acidic due to +I effect of the adjacent methyl group. Hence β′-hydrogen is relatively more acidic and is removed to give the alkene (ethene) by route a.

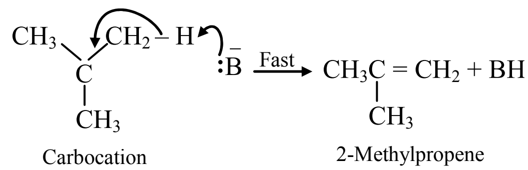
In elimination reactions steric effect also plays an important role. Thus, dehydrohalogenation of alkyl halide using the bulky base leads to the formation of terminal alkene as the major product.



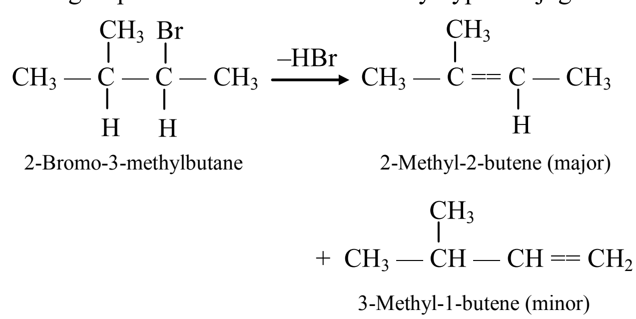
Unimolecular elimination reactions (E1) :

In these reactions the rate of elimination is dependent only on the concentration of the substrate and is independent of the concentration of the nucleophile and the reaction is of first order, (E1). Like S_N1 reaction the E1 reaction is also a two step process. The first step is the slow ionization of alkyl halide to give the carbocation. The second step involves the fast abstraction of a proton from the adjacent β-carbon atom giving rise to the formation of an alkene.

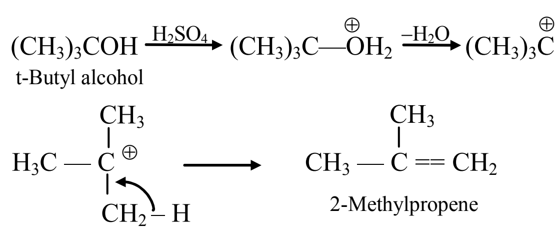




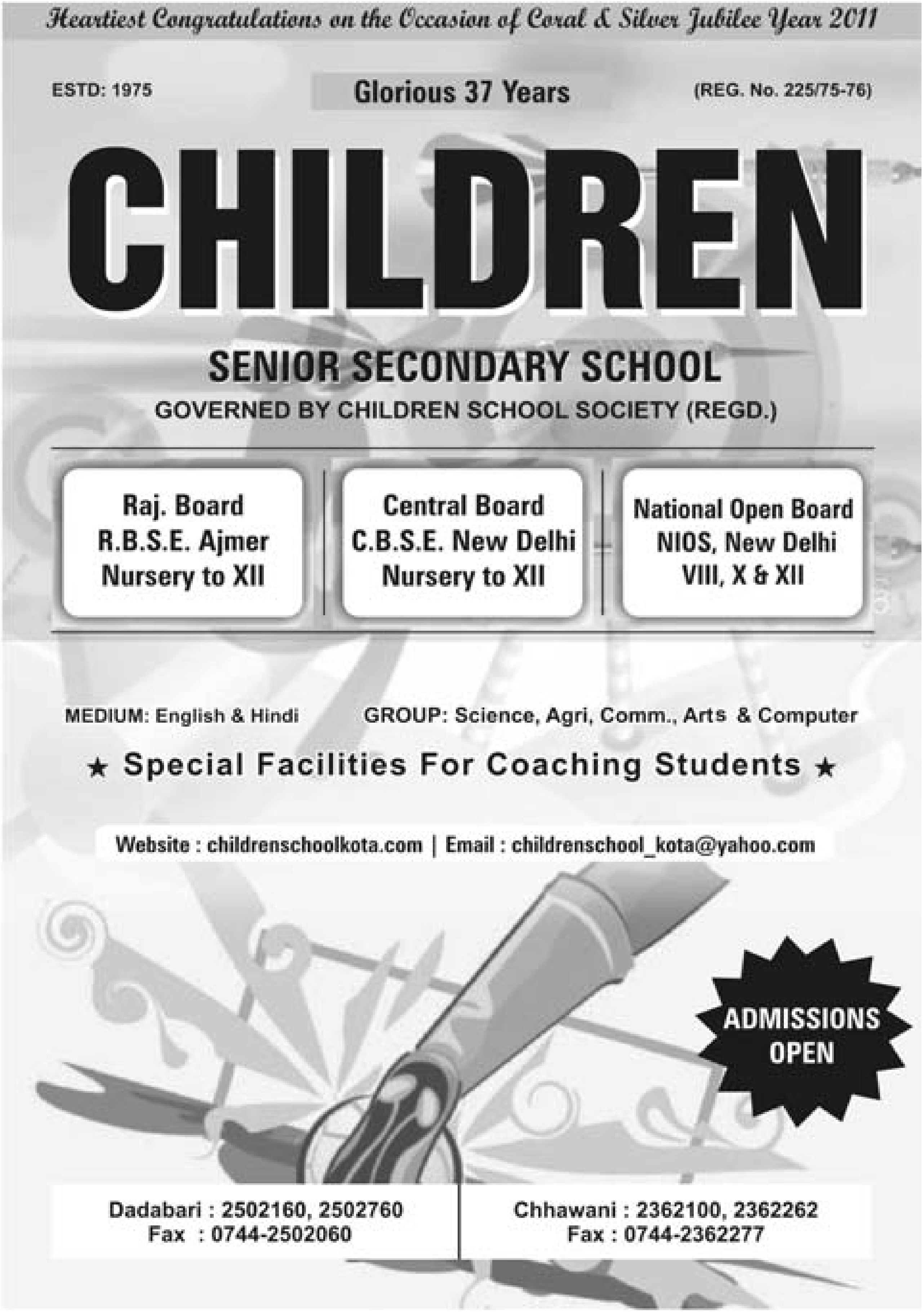
In case the substrate is such that more than one alkenes can be formed, that alkene will predominate which has larger number of alkyl groups on the double bonded carbon (this is as per Saytzeffs rule. This can be visualised since the substituted alkyl groups will stabilise the alkene by hyperconjugation.



The acid catalysed dehydration of alcohols also follows E1 mechanism.



In the E1 mechanism the rate of reaction is determined by the rate of formation of carbocation, which in turn depends on the stability of carbocation. Due to the formation of carbocation, these may undergo rearrangements. This has been experimentally confirmed.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KEY CONCEPT

Physical Chemistry Fundamentals

ENERGETICS

Thermodynamics deals with the transfer of heat between a chemical system and its surroundings when a reaction or phase change takes place within the system. The entire formulation of thermodynamics is based on two fundamental laws which have been established on the basis of experimental on the basis of experimental behaviour of macroscopic aggregates of matter, collected over a long period of time.

Since

First Law of thermodynamics

The internal energy of a system can be changed by transferring heat to/from the system from/to the surroundings. It can also be changed by doing the mechanical work on/by the system by/on the surroundings. These facts are represented in the form of the first law of thermodynamics as

$$dU = dq + dw \quad \text{or} \quad \Delta U = q + w$$

Since heat given to the system and work done on the system raise the internal energy of the system, these two operations are assigned positive values. The converse of the two operations, viz., heat given out and work done by the system are assigned negative values.

The expression of work done by/on a gaseous system is given by

$$dw = -\,p_{ext}\,dV$$

Where p_{ext} is the external pressure against which the volume gaseous system is changed by an amount dV. For a constant external pressure, we have

$$w = -\,nRT \ln (V_2/V_1)$$

where V_1 and V_2 are the initial and final volumes of the gaseous system.

If p_{ext} differs from the pressure of the gas by infinitesimal amount, the work is said to be carried out under reversible condition. In this case, the expression of work under constant temperature condition is given by

$$w = -\,nRT \ln (V_2/V_1)\ '$$

Note that for $V_2 > V_1$, there occurs an expansion of the gas. The work is done by the system on the surroundings and it carries a negative sign.

For $V_1 > V_2$, there occurs compression of the gaseous system. The work is done by the surroundings on the system and it carries a positive sign.

Internal energy and enthalpy

From the first law of thermodynamics, it can be shown that the heat transferred at constant volume changes the internal energy of the system, whereas that at constant pressure changes the enthalpy of the system .

$$\Delta U = nC_{v,m} (T_2 - T_1) \text{ ; and } \Delta H = nC_{p,m} (T_2 - T_1)$$

where $C_{v,m}$ and $C_{p,m}$ are the molar heat capacities at constant volume and constant pressure, respectively.

In the laboratory, the majority of chemical reactions are carried out under the condition of constant pressure, and thus the heat transferred in such a system is equal to the enthalpy change in a chemical reaction. Since the enthalpy of a system can also change due to the variation in temperature and pressure, it is, therefore essential that the reactants and products in a chemical reaction must have the same temperature and pressure.

Enthalpy change of a chemical equation

The enthalpy change of thermochemical equation is

$$\Delta H = \sum_{\text{(products)}} v_j H_{m,j} - \sum_{\text{(reactants)}} v_i H_{m,i}$$

where $H_{m,i}$ refers to the molar enthalpy of species i in the balanced chemical equation and v_i the corresponding stoichiometric coefficient. The unit of ΔH are kJ mol^{-1} .

Two types of reactions may be distinguished.

(a) **Exothermic reactions** For these ΔH is negative, which implies negative q_p and hence release of heat when reactants are converted into products. In this case

$$\Sigma H(\text{products}) < \Sigma H(\text{reactants})$$

(b) **Endothermic reactions** For these ΔH is is positive, which implies positive q_p and hence absorption of heat when reactants are converted into products. In this case

$$\Sigma H(\text{products}) > \Sigma H(\text{reactants})$$

Molar enthalpies of formations

It is not possible to determine the absolute value of enthalpy of a substance. However, based on the following convention, the relative values of standard molar enthalpies of formation (the term standard indicates of pressure of 1 bar) other substances can be determined.

The enthalpy of formation of every element in its stable states of aggregation at 1 bar and 25ºC is assigned a zero value.

For example, $\Delta_f H^o$ (graphite) = 0 $\Delta_f H^o$ (Br_2, 1) = 0 $\Delta_f H^o$ (S, rhombic) = 0 $\Delta_f H^o$ (H_2, g) = 0 and so on.]

The enthalpy change of a chemical equation can be computed by using the expression

$$\Delta_r H^o = \sum_{(products)} \Delta_f H_i^0 - \sum_{(reactors)} \Delta_f H_i^0$$

Hess's law of constant heat summation

Since the molar enthalpies of formation of reactants and products involved in a chemical equation have definite values, the enthalpy change of (or heat involved in) the chemical equation will have a definite value, irrespective of the fact whether the reaction is carried out in one step or more than one step. This fact is known as Hess's law of constant heat summation. For example,

(i) $C(graphite) + O_2((g) \rightarrow CO_2(g)$

$\Delta H_1 = -393.51 \text{ kJ mol}^{-1}$

(ii) $C(graphite) + \frac{1}{2} O_2(g) \rightarrow CO(g)$

$\Delta H_2 = -110.52 \text{ kJ mol}^{-1}$

$CO(g) + \frac{1}{2} O_2(g) \rightarrow CO_2(g)$

$\Delta H_3 = -282.99 \text{ kJ mol}^{-1}$

Obviously, $\Delta H_1 = \Delta H_2 + \Delta H_3$

Types of reactions and corresponding enthalpy changes

The enthalpy change in a reaction is suitable named according to the type of reaction in question. Two types of reaction are specifically defined as follows.

Enthalpy of formation: the enthalpy of combustion of a given substance is defined as the enthalpy change when 1 mole of a given substance is formed, starting from the elements in their stable states of aggregation. A few examples are

$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(1)$

$\Delta_f H^o(H_2O, 1) = -285.77 \text{ kJ mol}^{-1}$

$12C(graphite) + 11H_2(g) + \frac{11}{2} O_2(g) \rightarrow C_{12}H_{22}O_{11}(s)$

$\Delta_f H^o(C_{12}H_{22}O_{11},s) = -2218 \text{ kJ mol}^{-1}$

Enthalpy of Combustion: The enthalpy of combustion of a given substance is defined as the enthalpy change when one mole of this substance combines with requisite amount of oxygen to form products in their stable states of aggregation. A few examples are

$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(\ell)$

$\Delta_c H(CH_4, g) = -74.85 \text{ kJ mol}^{-1}$

$C_{12}H_{22}O_{11}(s) + 12O_2(g) \rightarrow 12CO_2(g) + 11H_2O(\ell)$

$\Delta_c H(C_{12}H_{22}O_{11}, s) = -5644 \text{ kJ mol}^{-1}$

Similarly, one can mane the enthalpy change based on the type of reaction. A few examples are

Enthalpy of fusion: $H_2O(s) \xrightarrow[273.15K]{} H_2O(\ell)$

$\Delta_{fus} H = 6 \text{ kJ mol}^{-1}$

Enthalpy of vaporization :

$H_2O(1) \xrightarrow[373.15K]{} H_2O(g)$

$\Delta_{vap} H = 40.6 \text{ kJ mol}^{-1}$

Enthalpy of sublimation : $I_2(s) \rightarrow I_2(g)$

$\Delta_{sub} H = 63.4 \text{ kJ mol}^{-1}$

Enthalpy of transition : $C(graphite) \rightarrow C(diamond)$

$\Delta_{trs} H = 1.90 \text{ kJ mol}^{-1}$

Enthalpy of neutralization : $H^+(aq)+OH^-(aq) \rightarrow H_2O(\ell)$

$\Delta_{neut} H = -57.3 \text{ kJ mol}^{-1}$

Enthalpy of ionization : $HCN(aq) \rightarrow H^+(aq)+CN^-(aq)$

$\Delta_{ioniz} H = 45.17 \text{ kJ mol}^{-1}$

Relation between ΔH and ΔU of a chemical equation

Since, $H = U + pV$, we have

$\Delta H = \Delta U + \Delta(pV) = \Delta U + (\Delta v_g) RT$

where Δv_g is the change in the stoichiometric number of gaseous molecules in converting reactants to products and is given as

$$\Delta v_g = \sum_{(products)} v_{g,i} - \sum_{(reactants)} v_{g,i}$$

rFor a reaction involving condensed phases

$$\Delta H \simeq \Delta U$$

Bond Enthalpies: Bond enthalpy of a given bond is defined as the average enthalpies required to dissociate the said bond present in different gaseous compounds into free atoms in the gaseous phase. The bond enthalpy may be distinguished from bond dissociation enthalpy which is enthalpy required to dissociate a given bond of some specific molecule. It is possible to construct a table listing the average bond enthalpies of different types of bonds and with the help of this, one can estimate the enthalpy change of a chemical equation involving gaseous species. For example, for a reaction

$H_2(g) + Cl_2(g) \rightarrow 2HCl(g)$

we can write $\Delta H = +\varepsilon(H–H) + \varepsilon(Cl–Cl) - 2\varepsilon(H–Cl)$

Second Law of thermodynamics

The second law of thermodynamics identifies a state function, called the entropy, which provides a criterion for identifying reversible or irreversible nature of the given process undergone by a system. The entropy of the universe (system + surroundings) increases for irreversible processes whereas it remains constant for reversible processes.

The entropy function has been identified with the disorderliness of the system–larger the disorderliness, larger the entropy of the system. Foe example, for a substance in three states of matter we have

S(gaseous state) > > S(liquid state) > (solid state)

Expression of Entropy Function

For a system which involves transferring infinitesimal heat at constant temperature, the entropy change of the system is given by

$$dS = \frac{dq_{rev}}{T}$$

For finite heat transferred at constant temperature, we have

$$\Delta S = \frac{q_{rev}}{T}$$

For example, for a pure substance we have

$$\Delta_{vap}S = \frac{\Delta_{vap}H}{T_b} \text{ and } \Delta_{fus}S = \frac{\Delta_{fus}H}{T_m}$$

where the subscripts vap and fus represent vaporization and fusion, respectively.

Gibbs Function

Gibbs function (or energy) or simply free energy is defined as

$$G = H - TS$$

For a process occurring at constant T and P, the change in Gibbs function is given by

$$\Delta G = \Delta H - T\,\Delta S$$

For a process to be spontaneous, the value of ΔG is negative. For a nonspontaneous reaction, ΔG is positive. For a reaction at equilibrium, $\Delta G = 0$ and temperature at which the system occurs at equilibrium is given by

$$T_{eq} = \Delta H/\Delta S$$

Pressure-Volume Work

An ideal gas can undergo expansion of compression under isothermal or adiabatic conditions. The expansion ant compression may be carried out under reversible or irreversible conditions. We give below the expressions of p-V work under different conditions.

Isothermal p–V Work

In this case, temperature of the system remains constant, ie. $\Delta T = 0$

For irreversible condition: $w = -P_{ext}(V_2 - V_1)$

For reversible condition: $w = -nRT \ln(V_2/V_1)$

Adiabatic p–V Work

In this case, heat can neither enter to or leave from the system, i.e. q = 0. From first law of thermodynamics, it follows that

$$\Delta U = w$$

where ΔU is given by

$$\Delta U = C_v(T_2 - T_1)$$

For a gas undergoing adiabatic irreversible volume change, the expression of work is given by

$$w = -P_{ext}(V_2 - V_1)$$

For an ideal gas undergoing adiabatic reversible expansion/compression, we also have

$$pV^{\gamma} = \text{constant}$$

$$pT^{\gamma(1-\gamma)} = \text{constant}$$

and $$TV^{\gamma-1} = \text{constant}$$

here $\gamma = C_{p,m}/C_{v,m}$ The symbols $C_{p,m}$ and $C_{v,m}$ represent molar heat capacities at constant pressure and volume conditions, respectively.

For a monatomic ideal gas:

$C_{v,m} = (3/2)R$; $C_{p,m} = (5/2)R$; and $\gamma = 5/3$

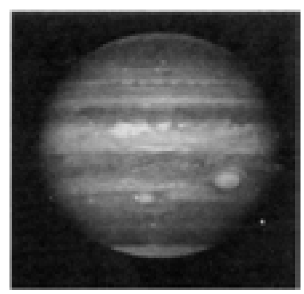
For a diatomic ideal gas:

$C_{v,m} = (5/2)R$; $C_{p,m} = (7/2)R$; and $\gamma = 7/5$

Birth of New Red Spot is the Thunderstorm on Jupiter

During the past few months, the astronomers have tracked an emerging second red spot on Jupiter, a growing rival about one-half the diameter of the planet's *Great Red Spot.* The Hubble Space Telescope has snapped the first detailed pictures of what some observers now call Red Spot Jr.

Astronomers at the Space Telescope Science Institute in Boltimore said this was the first time scientists have witnessed the birth of these huge oval spots, presumably a convective phenomenon like a powerful thunderstorm. The *Great spot* was already present when the observers first looked with telescope at the planet some 400 years ago. Red Spot Jr. appeared in near-infrared images to be as bright in Jupiter's cloudy atmosphere as its big companion. The size of Red Spot Jr. is half the size of its big companion. The scientists say the new storm might rise higher above the main cloud deck than the older spot.



In a New Light: Jupiter, with its second red spot, in a picture released by NASA.

Current observations, including Hubble pictures taken on May 12 and 18, show that the smaller red spot is drifting eastward in the Jovian Southern hemisphere and the *Great Red Spot* is moving westward. They should pass one another in early July. The pictures of the Red Spots are contrast-enhanced images taken in visible light and at near-infrared wave lengths. But the red spots, new and old, are really red.

CHEMISTRY

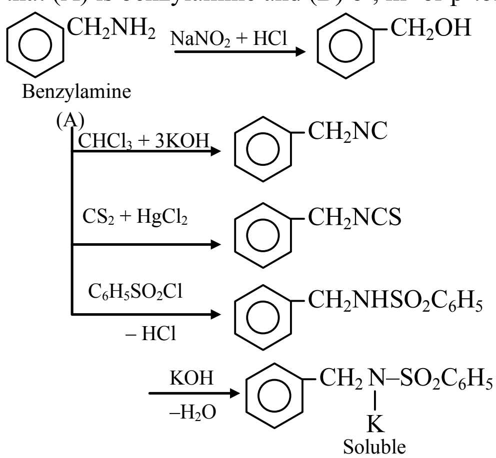
UNDERSTANDING

Organic Chemistry

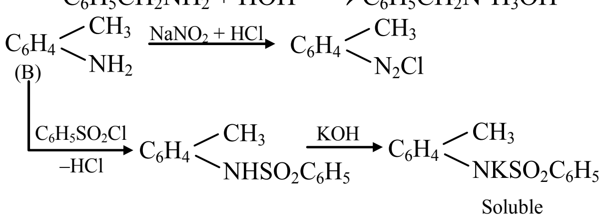
1. Two isomeric compounds (A) and (B) have the molecular formula C_7H_9N. (A) being soluble in water, the solution being alkaline to litmus It does not undergoes diazotization, but show carbylamine reaction and mustard oil reaction, it reacts with acetyl chloride and acetic anhydride. Its product with benzene sulphonyl chloride dissolves in KOH. (B) on the other hand, does not dissolve in water, but undergoes diazotization. Its product with $C_6H_5SO_2Cl$ dissolves in KOH. Its salt undergo hydrolysis in aqueous solution showing an acidic test. What are (A) and (B) ?

Sol. As both (A) and (B) give products with $C_6H_5SO_2Cl$, which are soluble in KOH, they contain $-NH_2$ group. (B) can be diazotized so contains $-NH_2$ in the nucleus. (A) cannot be diazotized, hence contains $-NH_2$ in the side chain. The number of carbon and hydrogen atoms also indicates aromatic character.

On the basis of above considerations we may show that (A) is benzylamine and (B) o⁻, m⁻ or p-toludine.



$C_6H_5CH_2NH_2 + HOH \longrightarrow C_6H_5CH_2N^+H_3OH^-$



2. A hydrocarbon (A) [C = 90.56%, V.D. = 53] was subjected to vigrous oxidation to give a dibasic acid (B). 0.10 g of (B) required 24.10 ml of 0.05 N NaOH for complete neutralization. When (B) was heated strongly with soda-lime it gave benzene. Identify (A) and (B) with proper reasoning and also give their structures.

Sol. Determination of empirical formula of (A) :

Element	%	Atomic wt.	Relative no. of atoms	Simplest ratio
C	90.56	12	$\frac{90.56}{12}$ = 7.55	$\frac{7.55}{7.55}$ = 1 or 4
H	9.44	1	$\frac{9.44}{1}$ = 9.44	$\frac{9.44}{7.55}$ = 1.25 or 5

The empirical formula of (A) = C_4H_5

Empirical formula weight = 48 + 5 = 53

Molecular weight = V.D. × 2 = 53 × 2 = 106

Hence, $n = \frac{\text{Molecular wt.}}{\text{Empirical wt.}} = \frac{106}{53} = 2$

Molecular formula = $2 \times C_4H_5 = C_8H_{10}$

The given equation may be outlined as follows :

$$\underset{(A)}{C_8H_{10}} \xrightarrow[6[O]]{\text{Vigrous oxidation}} \underset{(B)}{(HOOC)_2C_6H_4} + 2H_2O$$

Meq. of dicarboxylic acid = Meq. of NaOH

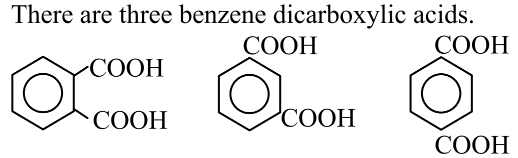
$$\frac{0.1 \times 1000}{E} = 24.1 \times 0.05$$

Equivalent of acid = 83

Molecular wt. = Basicity × Equivalent weight = 2 × 83 = 166

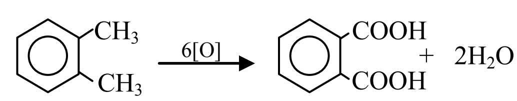
Since (B) on heating with soda-lime gives benzene, the C_6H_4 represents to benzene nucleus having two side chains, thus (B) is a benzene dicarboxylic acid.

There are three benzene dicarboxylic acids.

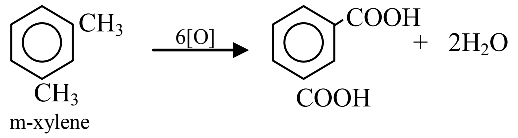


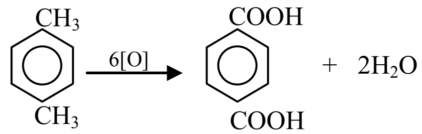
Phthalic acid Isophthalic acid Terphthalic acid

All the above three acids are obtained by the oxidation of respectively xylenes.

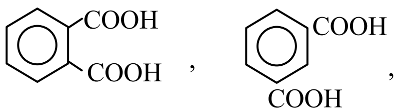


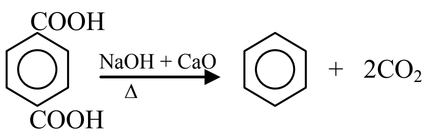
o-xylene



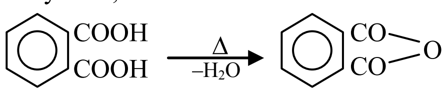


All the above three acids on heating with soda-lime yields only benzene.

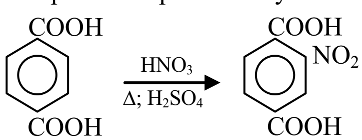




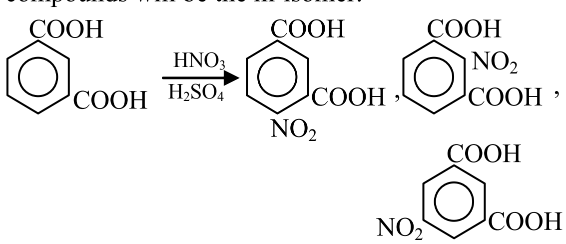
Of the three acids, one which on heating gives an anhydride, is o-isomer.



One acid which on nitration gives a mono nitro compounds is p-dicarboxylic acid.



One acid which on nitration gives three mono nitro compounds will be the m-isomer.



3. An organic compound (A), C_4H_9Cl, on reacting with aqueous KOH gives (B) and on reaction with alcoholic KOH gives (C) which is also formed on passing vapours of (B) over heated copper. The compound (C) readily decolourises bromine water. Ozonolysis of (C) gives two compounds (D) and (E). Compound (D) reacts with NH_2OH to gives (F) and the compound (E) reacts with NaOH to give an alcohol (G) and sodium salt (H) of an acid. (D) can also be prepared from propyne on treatment with water in presence of Hg^{++} and H_2SO_4. Identify (A) to (H) with proper reasoning.

Sol.
$$\underset{\text{(Alkyl halide)}}{\underset{(A)}{C_4H_9Cl}} \xrightarrow[\Delta;\ -KCl]{\text{Alc. KOH}} \underset{\text{(Alkene)}}{\underset{(C)}{C_4H_8}}$$
$$\xrightarrow[\Delta;\ -KCl]{\text{Aq.KOH}} \underset{\text{(Alcohol)}}{\underset{(B)}{C_4H_9OH}} \xrightarrow[\Delta;\ -H_2O]{\text{Cu}} (C)$$

We know that p-alcohol on heating with Cu gives aldehyde while s-alcohol under similar conditions gives ketone. Thus, (B) is a t-alcohol because it, on heating with Cu gives an alkene (C). Since a t-alcohol is obtained by the hydrolysis of a t-alkyl halide, hence (A) is t-butyl chloride.

$$(A) = CH_3-\overset{\overset{\displaystyle Cl}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-CH_3 \text{ and } (B) = CH_3-\overset{\overset{\displaystyle OH}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-CH_3$$

The alkene (C) on ozonolysis gives (D) and (E), hence (C) is not symmetrical alkene. In these compound (E) gives Cannizaro's reaction with NaOH. So, (E) is an aldehyde which does not contain α–H atom. Hence it is HCHO. Compound (D) can also be prepared by the hydration of propyne in the presence of acidic solution and Hg^{++}.

$$CH_3-C\equiv CH + H_2O \xrightarrow[H^+]{Hg^{++}} CH_3-\underset{\underset{\displaystyle OH}{|}}{C}=CH_2$$
$$\longrightarrow \underset{(D)}{CH_3-\underset{\underset{\displaystyle O}{\|}}{C}-CH_3}$$

Hence (D) is acetone and (E) is formaldehyde. Therefore, alkene (C) is 2-methyl propene. $(CH_3)_2–C=CH_2$

(D) reacts with hydroxyl amine (NH_2OH) to form oxime (F).

$$\underset{(D)}{\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C=}\boxed{O + H_2}\,NOH \xrightarrow{-H_2O} \underset{(F)}{\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C=NOH}$$

$$(B) = CH_3-\overset{\overset{\displaystyle OH}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-CH_3 \text{ and } (A) = CH_3-\overset{\overset{\displaystyle Cl}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-CH_3$$

Reactions :

$$\underset{(A)}{CH_3-\overset{\overset{\displaystyle Cl}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-CH_3} \xrightarrow[\Delta;-KCl]{\text{Aq.KOH}} \underset{(B)}{CH_3-\overset{\overset{\displaystyle OH}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-CH_3}$$

$$\xrightarrow[-H_2O]{Cu/300°C} \underset{(C)}{CH_3-\underset{\underset{\displaystyle CH_3}{|}}{C}=CH_2} + H_2O$$

$\xrightarrow[-KCl;\ -H_2O]{Alc.KOH/\Delta} CH_3-\underset{\underset{CH_3}{|}}{C}=CH_2$
(C)

$CH_3-\underset{\underset{CH_3}{|}}{C}=CH_2 \xrightarrow[(II)\ H_2O/Zn]{(I)\ O_3} \begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C=O + H-\overset{\overset{O}{||}}{C}-H$
(C) (D) (E)

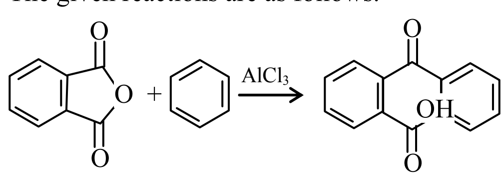
$\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C=O + H_2NOH \xrightarrow[-H_2O]{\Delta} \begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C=NOH$
(D) (F)

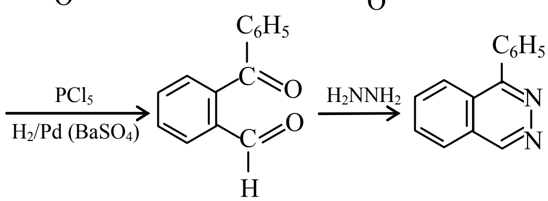
$\underset{(E)}{2HCHO} + NaOH \rightarrow \underset{(G)}{CH_3OH} + \underset{(H)}{HCOONa}$

$CH_3-C\equiv CH + H_2O \xrightarrow[H^+]{Hg^{++}} CH_3-\overset{\overset{O}{||}}{C}-CH_3$
(D)

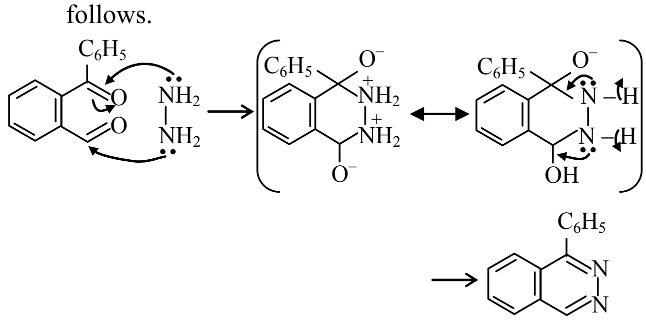
4. An organic compound A, $C_8H_4O_3$, in dry benzene in the presence of anhydrous $AlCl_3$ gives compound B. The compound B on treatment with PCl_5 followed by reaction with $H_2/Pd(BaSO_4)$ gives compound C, which on reaction with hydrazine gives a cyclised compound $D(C_{14}H_{10}N_2)$. Identify A, B, C and D. Explain the formation of D from C.

Sol. The given reactions are as follows.



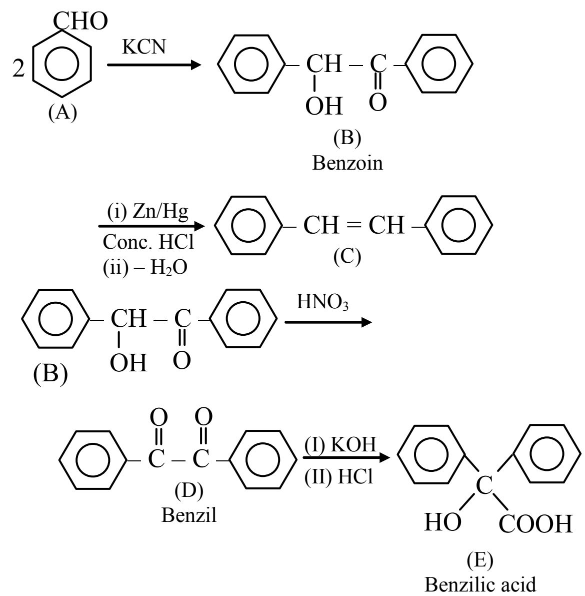


The formation of D from C may be explained as follows.



5. An organic compound (A), C_7H_6O, gives positive test with Tollen's reagent, on treatment with alcoholic potassium cyanide, (A) yields the compound (B), $C_{14}H_{12}O_2$. Compound (B) on reduction with Zn – Hg and conc. HCl and dehydration gives an unsaturated compound (C), which adds one mol of bromine. The compound (B) can be oxidised with HNO_3 to a compound (D), having molecular formula $C_{14}H_{10}O_2$. Compound (D) on heating with KOH undergoes rearrangement and subsequent acidification of rearranged products yields an acidic compound (E), $C_{14}H_{12}O_3$. Identify compounds (A) to (E) giving the reactions involved.

Sol.



Interesting Facts

- Roy J. Plunkett of New Carlisle, Ohio invented Teflon in 1938.
- The chemical n-acetyl-cysteine found in raw eggs is proven to help hangovers.
- Beetles are the strongest animals on Earth relative to their size. A rhinoceros beetle can carry 850 times its own weight in its back.
- Muffins spelled backwards is sniffum.
- The plague in Zurich killed 3,700 of the cities 6,000 inhabitants in 1567.
- Fish have no eyelids. They can't blink, wink or close their eyes to sleep.
- For centuries salt makers used conical wicker basket moulds called barrows to make their 'lumps' (like bricks only bigger!), which then had to be crushed before the salt could be used. The word 'lump' has passed into the English language. Workers had to 'lump' the salt and their job was known as 'lumping'.
- The first spacecraft to visit Venus was Mariner 2 in 1962. It was subsequently visited by many others (more than 20 in all). Including Pioneer Venus and the Soviet Venera 7 - the first spacecraft to land on another planet - and

Mathematical Challenges

Set 3

This section is designed to give IIT JEE aspirants a thorough grinding & exposure to variety of possible twists and turns of problems in mathematics that would be very helpful in facing IIT JEE. Each and every problem is well thought of in order to strengthen the concepts and we hope that this section would prove a rich resource for practicing challenging problems and enhancing the preparation level of IIT JEE aspirants.

By : Shailendra Maheshwari
Joint Director Academics, Career Point, Kota

Solutions will be published in next issue

1. Let $y = f(x)$ be a curve satisfying
$\frac{dy}{dx} - y \ln 2 = 2^{\sin x}(\cos x - 1)$. ln2, then
(A) y is bounded when $x \to \infty$
(B) $f(x) = 2^{\sin x} + c \,.\, 2^x$, where c is an arbitrary constant
(C) $y = 2^{\sin x}$, y is bounded when $x \to \infty$
(D) $f(x) = 2^{\text{sinx}}$ does not have any solution if y is not bounded.

2. In a right angled triangle the length of its hypotenuse is four times the length of the perpendicular drawn from its orthocentre on the hypotenuse. The acute angles of the triangle can be
(A) $\frac{\pi}{6}, \frac{\pi}{3}$ (B) $\frac{\pi}{8}, \frac{3\pi}{8}$
(C) $\frac{\pi}{6}, \frac{\pi}{4}$ (D) $\frac{\pi}{12}, \frac{5\pi}{12}$

3. Let $a, b \in R$ such that $0 < a < 1$ and $0 < b < 1$. The values of a and b such that the complex number $z_1 = -a + i$, $z_2 = -1 + bi$ and $z_3 = 0$ form an equilateral triangle are
(A) $2-\sqrt{3}, \sqrt{3}$ (B) $2-\sqrt{3}, 2-\sqrt{3}$
(C) $\sqrt{3}, 2-\sqrt{3}$ (D) None of these

4. If c_1 is a fixed circle and c_2 is a variable circle with fixed radius. The common transverse tangents to c_1 and c_2 are perpendicular to each other. The locus of the centre of variable circle is :
(A) circle (B) ellipse
(C) hyperbola (D) parabola

5. The length of the latus rectum of the parabola
$169\,\{(x-1)^2 + (y-3)^2\} = (5x - 12y + 17)^2$ is –
(A) $\frac{14}{13}$ (B) $\frac{56}{13}$ (C) $\frac{28}{13}$ (D) None

6. Evaluate : $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x}\, dx$

7. Find all the real values of a, for which the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ lie between the roots of equation
$$x^2 - 2(a+1)\,x + a(a-1) = 0$$

8. Given the base of a triangle and the sum of its sides prove that the locus of the centre of its incircle is an ellipse.

9. A bag contains 7 tickets marked with the number 0, 1, 2, 3, 4, 5, 6 respectively. A ticket is drawn and replaced. Then the chance that after 4 drawings the sum of the numbers drawn is 8, is –

10. A polynomial in x of degree greater than 3 leaves remainders 2, 1 and – 1 when divided by $(x-1)$, $(x+2)$ and $(x+1)$ respectively. What would be the remainder if the polynomial is divided by $(x^2 - 1)(x + 2)$?

Success Funda

ATTITUDE AND CONFIDENCE

Attitude and confidence level carries you forward or backward. It is the attitude that matters. It is as important as your expertise or knowledge. Don't ever let negative feelings overtake you or come near you. Always say to you, "I shall do my best and succeed," Any negative suggestions adversely affect attitudes. Remind yourself that if you do not succeed expectations sometimes tempt you to adopt unfair means. Keep in mind that, occasionally, it is natural to fail in any venture but never as a human being.

MATHEMATICAL CHALLENGES

SOLUTION FOR JUNE ISSUE (SET # 2)

1. 1^{st} box can be filled in 4 ways.
Next each box can be filled in 3 ways (except the ball of colour in previous box).
Hence the required no. of ways = $4 \times 3^5 = 972$

2. Given $|A| \neq 0$; $AA^{-1} = I \Rightarrow (AA^{-1})^T = I^T$
$(A^{-1})^T A^T = I$ (as A is symmetric)
$(A^{-1})^T A = I$
so by the definition of inverse $A^{-1} = (A^{-1})^T$
Hence A^{-1} is also symmetric.

3. The normal to hyperbola at the point
P(a sec θ, b tan θ) is
$$ax \cos\theta + by \cot\theta = a^2 + b^2$$
If it passes through (h, k) then
$$a\,h \cos\theta + b\,k \cot\theta = a^2 + b^2 \quad ...(1)$$
Let $z = e^{i\theta} = \cos\theta + i\sin\theta$ then put $\cos\theta = \dfrac{z^2+1}{2z}$
and $\cot\theta = i\left(\dfrac{z^2+1}{z^2-1}\right)$ in the equation (1).
$$ahz^4 + 2(i\,bk - (a^2+b^2))\,z^3 + 2(i\,bk + (a^2+b^2))z - ah = 0$$
z_1, z_2, z_3, z_4 are its four solutions so
$\Sigma z_1z_2 = 0 = \Sigma\, e^{i(\theta_1+\theta_2)} = 0$
$\Sigma(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)) = 0$
Hence $\Sigma\cos(\theta_1+\theta_2) = \Sigma\sin(\theta_1+\theta_2) = 0$

4. Planes are $-x - 2y - 2z + 9 = 0$ ....(1)
and $4x - 3y + 12z + 13 = 0$ ...(2)
The plane bisecting the angle b/w these planes containing origin is
$$\frac{-x-2y-2z+9}{3} = + \frac{4x-3y+12z+13}{13}$$
i.e. $25x + 17y + 62z - 78 = 0$ ...(3)
If θ be the angle between (1) & (3) then
$$\cos\theta = \frac{61}{\sqrt{4758}}$$
$$\Rightarrow \tan\theta = \frac{\sqrt{1037}}{61} < 1$$
Hence plane given by (3) is bisecting the acute angle between given two planes also. Hence the conclusion holds true.

5. Let $I_2 = \int_{f(a)}^{f(b)} ((f^{-1}(y))^2 - a^2)\,dy$
Let $f^{-1}(y) = x$
$\Rightarrow$ $f(x) = y$
$$I_2 = \int_a^b (x^2 - a^2) f'(x)\,dx$$
Using by parts
$$I_2 = \left((x^2-a^2)f(x)\right)_a^b - \int_a^b 2x\,f(x)\,dx$$
$$= (b^2 - a^2)\,f(b) - \int_a^b 2x\,f(x)\,dx$$
$$= \int_a^b 2x\,f(b)\,dx - \int_a^b 2x\,f(x)\,dx$$
$$= \int_a^b 2x\,(f(b) - f(x))\,dx$$
Hence $\dfrac{I_1}{I_2} = \dfrac{1}{2}$

6. $y + \dfrac{1}{y} = 2$
$\Rightarrow y = 1$
$x + \dfrac{1}{x} = \sqrt{\sqrt{5}+2}$
$\Rightarrow x^2 + \dfrac{1}{x^2}$
$= (\sqrt{5} + 2) - 2 = \sqrt{5}$

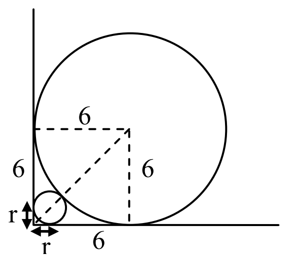
$x^4 + \frac{1}{x^4} = 5 - 2$

$\Rightarrow x^8 + \frac{1}{x^8} = 9 - 2$

$\Rightarrow x^{16} + \frac{1}{x^{16}} = 49 - 2$

$\Rightarrow 47 + 1 + 1 = 49$

7. Let the radius of S_2 is r



$\sqrt{2}\, r + r + 6 = \sqrt{2}\, 6$

$r = 6\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

$= 6(3 - 2\sqrt{2})$

$= 18 - 12\sqrt{2}$

8. $S_1 = 2 + 4 + 6 + ... + 120$

$= \frac{60}{2}(2 + 120)$

$= 30 \times 122 = 3660$

$S_2 = 7 + 14 + 21 + ... + 119$

$= \frac{17}{2}(7 + 119)$

$= 17 \times 63 = 1071$

$S_3 = 14 + 28 + ... + 102$

$= \frac{8}{2}(14 + 112)$

$= 4 \times 126 = 504$

Ans. $= \frac{120 \times 121}{2} - 3660 - 1071 + 504$

$= 7260 - 4731 + 504$

$= 2529 + 504$

$= 3033$

9. Here $F(x)$ is even function

so $f(x) = F(-x) = F(x)$

$\Rightarrow f(-x) = f(x)$

$g(x) = -F(x) = -f(x) = -f(-x)$

$h(x) = -F(-x) = -F(x) = -f(x)$

Ans. (C)

10. $f(x) + h(x) = f(x) - f(x) = 0$

$g(x) - h(x) = -f(x) + f(x) = 0$

$F(x) + f(x) \neq 0$

$f(x) - g(x) = f(x) + f(x) \neq 0$

Ans. (B)

What is mercury poisoning?

CHEMICAL DANGER Too much mercury can make you sick, but sometimes the symptoms are hard to distinguish from other illnesses.

What's mercury?

There are three kinds of mercury. Depending on what the exposure is, you could have different symptoms and disease states.

Elemental, or metal mercury, is found in thermometers. The problem with that is the inhalation of fumes that come off that mercury. Playing with it and ingesting it is not as toxic. That kind of mercury causes significant amounts of neurological damage. As the exposure gets longer, there may be additional changes in the bone marrow that affect the ability to produce blood cells, infertility and problems with heart rhythm.

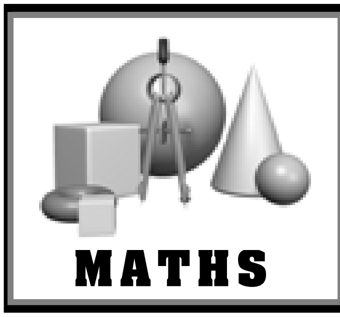
Mercury salts, which are basically industrial, if you breathe in or ingest them, gravitate more toward the kidney and not so much the nervous system.

- The organic mercury is what gets into the food chain. It's put into the water by chemical plants that are manufacturing things and they get into shellfish and fish, or elemental mercury that gets into the water is changed into organic mercury by sea life; we eat fish or shellfish and we get mercury exposure. That organic mercury acts very similarly to the elemental form. It affects a lot of nervous system damage. If a woman is pregnant, this can also cause birth defects and loss of the fetus if the levels get high enough.

Is mercury something we need in our diets, or is no amount nutritionally safe or necessary?

No level is normal. Zero is normal. It doesn't have a specific reason to be in our body. As long as we live on this Earth, because it's in Earth's crust and in the atmosphere, we're going to be exposed. But there is no specific function for that metal in our body.

The issue is one of looking at the total body burden: How much mercury is in the body and what's known to be a normal background? Theoretically, there's going to be a baseline level, a general population average, but depending on where you live, that level may be higher or lower. If you live near a coast,



Students' Forum

Expert's Solution for Question asked by IIT-JEE Aspirants

1. Suppose a function $f(x)$ satisfies the following conditions $f(x+y) = \dfrac{f(x)+f(y)}{1+f(x).f(y)}$ $\forall$ x, y and $f'(0) = 1$. Also $-1 < f(x) < 1$ for all $x \in R$, then find the set of values of x where $f(x)$ is differentiable and also find the value of $\lim\limits_{x\to\infty} [f(x)]^x$.

Sol. First put $x = 0$, $y = 0 \Rightarrow f(0) = 0$

Now, $f'(x) = \lim\limits_{x\to 0} \dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{x\to 0} \frac{\dfrac{f(x)+f(h)}{1+f(x).f(h)} - f(x)}{h}$$

$$= \lim_{x\to 0}\left\{\frac{f(h)-f(0)}{h-0}\right\}\left[\frac{1-\{f(x)\}^2}{-1+f(x).f(h)}\right]$$

$= 1 - \{f^2(x)\}$

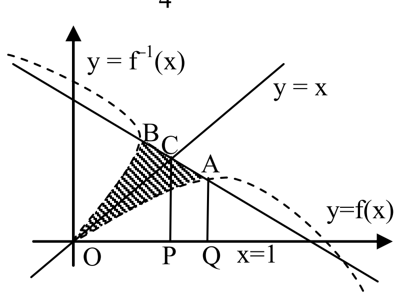
integrating we get $\dfrac{1}{2}\ln\left[\dfrac{1+f(x)}{1-f(x)}\right] = x + c$

$$\Rightarrow f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

clearly $f(x)$ is differentiable for all $x \in R$.

$$\lim_{x\to\infty}[f(x)]^x = \lim_{x\to\infty}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^x = e^{\lim\limits_{x\to\infty}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)} x = 1$$

2. If A be the area bounded by $y = f(x)$, $y = f^{-1}(x)$ and the line $4x + 4y - 5 = 0$ where $f(x)$ is a polynomial of 2^{nd} degree passing through the origin and having maximum value of $\dfrac{1}{4}$ at $x = 1$, then find A.



Sol. Let $f(x) = ax^2 + bx$

given that $\dfrac{1}{4} = a + b$ ...(1)

$0 = 2a + b$ ...(2)

from (1) and (2)

$a = -\dfrac{1}{4}$, $b = \dfrac{1}{2}$

$f(x) = \dfrac{2x - x^2}{4}$

since $4x + 4y - 5 = 0$ passes through $A\left(1, \dfrac{1}{4}\right)$ and $B\left(\dfrac{1}{4}, 1\right)$ so area bounded is OAB = 2OAC

= 2[ar(OCP) + ar(CAQP) – ar(OAQ)]

$$= 2\left[\frac{1}{2}\times\frac{5}{8}\times\frac{5}{8} + \frac{1}{2}\left(\frac{5}{8}+\frac{1}{4}\right)\frac{3}{8} - \int_0^1 \frac{2x-x^2}{4}dx\right]$$

$= \dfrac{37}{96}$ (unit)^2

3. Let f be a polynomial function such that $f(x) . f(y) + 2 = f(x) + f(y) + f(xy)$ $\forall$ $x \in R^+$, $y \in R^+ \cup \{0\}$ and $f(x)$ is one one $\forall$ $x \in R^+$ with $f(0) = 1$, $f'(1) = 2$, then find the area bounded between the curve $y = x^2$ and $y = g(x)$ where $g(x) = \dfrac{2}{f(x)}$ and x-axis and also find the no. of points of nondifferentiability of $h(x) = \min\{g(x), x^2, |1 - |x||\}$

Sol. Let $f(x) . f(y) + 2 = f(x) + f(y) + f(xy)$ ....(1)

putting $x = y = 1$

$f(1) . f(1) + 2 = 3f(1) \Rightarrow f(1) = 2, 1$

f is given one-one and $f(0) = 1$

$\Rightarrow f(1) = 2$

replacing y by $\dfrac{1}{x}$ in (1) than

$$f(x) . f\left(\frac{1}{x}\right) + 2 = f(x) + f\left(\frac{1}{x}\right) + 2$$

$\Rightarrow f(x) = 1 + x^n$

also $f'(1) = 2 \Rightarrow n = 2$

$\Rightarrow f(x) = 1 + x^2$

Now to find the area,

$$\text{Area} = 2\int_0^1\left(\frac{2}{1+x^2} - x^2\right)dx = 2\left(\frac{2\pi}{4} - \frac{1}{3}\right) = \pi - \frac{2}{3}$$

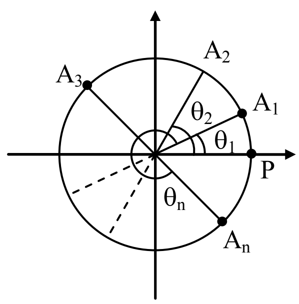
clearly by graph you can find there is 6 points of non differentiability.

4. Given a point P on the circumference of the circle $|z| = 1$, and vertices $A_1, A_2, ..., A_n$ of an inscribed regular polygon of n sides. Prove using complex numbers that

$(PA_1)^2 + (PA_2)^2 + ... + (PA_n)^2$ is a constant.

Sol. Without loss of generality we can take P as 1 + 0i.

i.e., $P \equiv C$ is 0



Let $A_r \equiv C$ is θ_r, r = 1, 2, ..., n.

$PA_r = |\text{Cis } \theta_r - \text{Cis } 0| = |(\cos\theta_r - 1) + i(\sin\theta_r)|$

$PA_r^2 = (\cos \theta_r - 1)^2 + (\sin\theta_r)^2$

$= 2 - 2\cos \theta_r$

$$\Rightarrow \sum_{r=1}^{n}(PA_r)^2 = 2n - 2\sum_{r=1}^{n}\cos\theta_r$$

$$\text{Now, } \sum_{r=1}^{n}\cos\theta_r = \text{Re}\left[\sum_{r=1}^{n} Cis\theta_r\right]$$

$$= \text{Re}[e^{i\theta_1} + e^{i\theta_2} + ... + e^{i\theta_n}]$$

$$= \text{Re}\left[\frac{e^{i\theta_1}\left(1-\left(e^{i\frac{2\pi}{n}}\right)^n\right)}{1-e^{i\frac{2\pi}{n}}}\right]$$

$$\because \theta_2 - \theta_1 = \theta_3 - \theta_2 = ... = \theta_n - \theta_{n-1} = \frac{2\pi}{n}$$

$$= \text{Re}\left[\frac{e^{i\theta_1}(1-1)}{1-e^{i\frac{2\pi}{n}}}\right] = 0$$

$$\text{Hence, } \sum_{r=1}^{n}(PA_r)^2 = 2n = \text{constant.}$$

5. Find the set of values of 'a' for which minimum value of $x^3 - 6ax^2 + 9a^2x + 7$, $x \in [-1, 2]$ is 3.

Sol. Let $f(x) = x^3 - 6ax^2 + 9a^2x + 7$

$a \neq 0$, otherwise $f(x) = x^3 + 7$, which is always increasing and hence $\min f = f(-1) = 6 \neq 3$.

Now $f'(x) = 3x^2 - 12ax + 9a^2 = 0$ for stationary points

$\Rightarrow x = a, 3a$

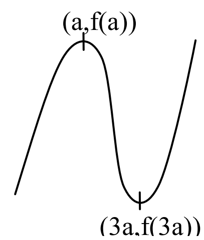
CaseI : $a > 0$

$\Rightarrow -1$ is always in the left of a.

Case I. (a) : $2 \leq a$, then

$3 = \min f = f(-1) = -1 - 6a - 9a^2 + 7$

$\Rightarrow 3a^2 + 2a - 1 = 0$, no admissible value of a is obtained.



Case I. (b) : $-1 < a < 2 < 3a$

i.e., $\frac{2}{3} < a < 2$, then

$3 = \min f = \min\{f(-1), f(2)\}$

$= \min \{-1 - 6a - 9a^2 + 7, 8 - 24a + 18a^2 + 7\}$

$= -1 - 6a - 9a^2 + 7$

as $-1 - 6a - 9a^2 + 7 < 8 - 24a + 18a^2 + 7$

i.e., $3a^2 - 2a + 1 > 0$, which is true

Hence $3 = -1 - 6a - 9a^2 + 7$

$\Rightarrow a = -1$ or $\frac{1}{3}$, none of which is possible.

Case I(c) : $3a \leq 2$

$\Rightarrow 3 = \min f = \min\{f(-1), f(3a)\}$

$= \{-1 - 6a - 9a^2 + 7, 18a^3 + 7\}$

$= -1 - 6a - 9a^2 + 7,$

as $18a^3 + 77 - 1 - 6a - 9a^2 + 7$

i.e., $18a^3 + 9a^2 + 6a + 170$

which is true as $a > 0$. Hence $a = -1$ or $\frac{1}{3}$,

in which $a = \frac{1}{3}$ is permissible.

Case II : $a < 0$

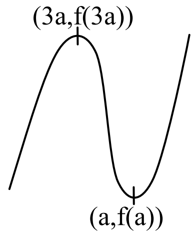
$\Rightarrow 2$ is always in the right of a

Case II (a) $a \leq -1$

$\Rightarrow 3 = \min f = f(-1)$

$\Rightarrow a = -1$, as $a = \frac{1}{3}$

Hence $a = -1$ is one possibility



Case II (b) $3a \le -1 < a$

$\Rightarrow -1$, as $a = \dfrac{1}{3}$

$\Rightarrow 3 = \min f = f(a) = a^3 - 6a^3 + 9a^3 + 7$

$\Rightarrow\ 4a^3 = -4 \Rightarrow a = -1$, not possible

Case II(c) $-1 < 3a \Rightarrow 3 = \min f = \min \{f(-1), f(a)\}$

$= \min \{-1 - 6a - 9a^2 + 7, 4a^3 + 7\}$

$= 4a^3 + 7,$

as $4a^3 + 7 < -1 - 6a - 9a^2 + 7$

as $(a + 1)^2 (4a + 1) < 0$. Hence $a = -1$, not possible

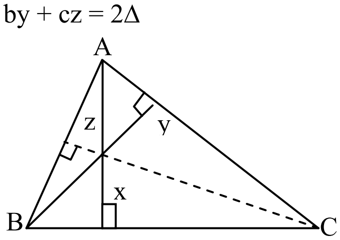
$\Rightarrow$ Hence $a = -1$ or $a = 1/3$

6. Find the point inside a triangle from which the sum of the squares of distance to the three side is minimum. Find also the minimum value of the sum of squares of distance.

Sol. If a, b, c are the lengths of the sides of the Δ and x, y, z are length of perpendicular from the points on the sides BC, CA and AB respectively, we have to minimise : $\Delta = x^2 + y^2 + z^2$

we have, $\dfrac{1}{2}ax + \dfrac{1}{2}by + \dfrac{1}{2}cz = \Delta$

$\Rightarrow ax + by + cz = 2\Delta$



where Δ is the area of ΔABC.

We have the identity :

$\Rightarrow (x^2 + y^2 + z^2)(a^2 + b^2 + c^2) - (ax + by + cz)^2$

$= (ax - by)^2 + (by - cz)^2 + (cz - ax)^2$

$\Rightarrow (x^2 + y^2 + z^2)(a^2 + b^2 + c^2) \ge (ax + by + cz)^2$

$\Rightarrow (x^2 + y^2 + z^2)(a^2 + b^2 + c^2 \ge 4\Delta^2$

$\Rightarrow x^2 + y^2 + z^{-2} \ge \dfrac{4\Delta^2}{a^2 + b^2 + c^2}$

Equality holds only when

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{ax + by + cz}{a^2 + b^2 + c^2} = \frac{2\Delta}{a^2 + b^2 + c^2}$$

$\therefore$ The minimum value of Δ is ;

$$\frac{4\Delta^2}{a^2 + b^2 + c^2} = \frac{4(s-a)(s-b)(s-c)s}{a^2 + b^2 + c^2}$$

XTRA FUNDA

Chemistry Facts

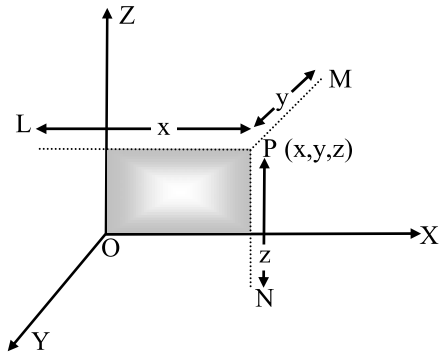
- After firing 5 billion billion zinc ions at a speed of 18,460 miles per second (30,000 kilometers per second) at lead, the German scientists at Darmstadt, Germany created a single atom of 112 protons (ununbium) that survived for one third (1/3) of a millisecond.
- If an electric current is passed through a solution or molten salt (the electrolyte), ions will migrate to the electrodes: positive ions (cations) to the negative electrode (cathode) and negative ions (anions) to the positive electrodes (anions).
- The positron was discovered in 1932 by the U.S. physicist Carl Anderson at California Institute of Technology, United States.
- Fritz Haber developed chlorine gas for use by the Germans in World War I. (Unable to live with his, his wife commited suicide in 1915).
- The flatulence of a single sheep could power a small truck for 25 miles (40 kilometers) a day. The digestive process produces methane gas, which can be burned as fuel.
- Cesiums has a diameter of 0.0000002 inches (0.0000005 millimeter).
- Hydrogen atoms with no neutrons make up 99.985% percent of all hydrogen atoms. The remaining 0.015% percent contain one neutron.
- The very first shell of an atom (innermost) can hold only up to two electrons.
- The element with the lowest boiling point is also helium at -452.07 degrees Fahrenheit (-268.93 degrees Celsius
- The word "atom" comes from the Greek word atomos, meaning "uncut."
- The first and relatively pure atom of tantalum was produced by von Bolton in 1907.

MATHS

3-DIMENSIONAL GEOMETRY

Mathematics Fundamentals

Coordinates of a point :



x-coordinate = perpendicular distance of P from yz-plane

y-coordinate = perpendicular distance of P from zx-plane

z-coordinate = perpendicular distance of P from xy-plane

Coordinates of a point on the coordinate planes and axes:

yz-plane	:	$x = 0$
zx-plane	:	$y = 0$
xy-plane	:	$z = 0$
x-axis	:	$y = 0, z = 0$
y-axis	:	$y = 0, x = 0$
z-axis	:	$x = 0, y = 0$

Distance between two points :

If P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are two points, then distance between them

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Coordinates of division point :

Coordinates of the point dividing the line joining two points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) in the ratio $m_1 : m_2$ are

(i) in case of internal division

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

(ii) in case of external division

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

Note: When m_1, m_2 are in opposite sign, then division will be external.

Coordinates of the midpoint:

When division point is the mid-point of PQ, then ration will be 1 : 1; hence coordinates of the mid-point of PQ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Coordinates of the general point :

The coordinates of any point lying on the line joining points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) may be taken as

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}, \frac{kz_2 + z_1}{k+1}\right)$$

which divides PQ in the ratio k : 1. This is called general point on the line PQ.

Division by coordinate planes :

The ratios in which the line segment PQ joining P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) is divided by coordinate planes are as follows :

(i) by yz-plane	:	$-x_1/x_2$ ratio
(ii) by zx-plane	:	$-y_1/y_2$ ratio
(iii) by xy-plane	:	$-z_1/z_2$ ratio

Coordinates of the centroid :

(i) If (x_1, y_1, z_1); (x_2, y_2, z_2) and (x_3, y_3, z_3) are vertices of a triangle then coordinates of its centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

(ii) If (x_r, y_r, z_r); r = 1, 2, 3, 4 are vertices of a tetrahedron, then coordinates of its centroid are

$$\left(\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{y_1 + y_2 + y_3 + y_4}{4}, \frac{z_1 + z_2 + z_3 + z_4}{4}\right)$$

Direction cosines of a line [Dc's] :

The cosines of the angles made by a line with positive direction of coordinate axes are called the direction cosines of that line.

Let α, β, γ be the angles made by a line AB with positive direction of coordinate axes then cos α, cos β, cos γ are the direction cosines of AB which are generally denoted by l, m, n. Hence

$$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$$

x-axis makes 0º, 90º and 90º angles with three coordinate axes, so its direction cosines are cos 0º, cos 90º, cos 90º i.e. 1, 0, 0. Similarly direction cosines of y-axis and z-axis are 0, 1, 0 and 0, 0, 1 respectively. Hence

dc's of x-axis = 1, 0, 0

dc's of y-axis = 0, 1, 0

dc's of z-axis = 0, 0, 1

Relation between dc's

$\therefore\ l^2 + m^2 + n^2 = 1$

Direction ratios of a line [DR's] :

Three numbers which are proportional to the direction cosines of a line are called the direction ratios of that line. If a, b, c are such numbers which are proportional to the direction cosines l, m, n of a line then a, b, c are direction ratios of the line. Hence

$$\Rightarrow\ l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}},$$

$$m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Direction cosines of a line joining two points :

Let ≡ (x_1, y_1, z_1) and Q ≡ (x_2, y_2, z_2); then

(i) dr's of PQ : $(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

(ii)dc's of PQ : $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$

i.e., $\frac{x_2 - x_1}{\sqrt{\Sigma(x_2 - x_1)^2}}, \frac{y_2 - y_1}{\sqrt{\Sigma(x_2 - x_1)^2}}, \frac{z_2 - z_1}{\sqrt{\Sigma(x_2 - x_1)^2}}$

Angle between two lines :

Case I. When dc's of the lines are given

If l_1, m_1, and l_2,m_2 n_2 are dc's of given two lines, then the angle θ between them is given by

- $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
- $\sin\theta = \sqrt{(\ell_1 m_2 - \ell_2 m_1)^2 + (m_1 n_2 - m_2 n_1)^2 + (n_1 \ell_2 - n_2 \ell_1)^2}$

The value of sin θ can easily be obtained by the following form :

$$\sin\theta = \sqrt{\begin{vmatrix} \ell_1 & m_1 \\ \ell_2 & m_2 \end{vmatrix}^2 + \begin{vmatrix} m_1 & n_1 \\ m_2 & n_2 \end{vmatrix}^2 + \begin{vmatrix} n_1 & \ell_1 \\ n_2 & \ell_2 \end{vmatrix}^2}$$

Case II. When dr's of the lines are given

If a_1, b_1, c_1 and a_2, b_2, c_2 are dr's of given two lines, then the angle θ between them is given by

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\sin\theta = \frac{\sqrt{\Sigma(a_1 b_2 - a_2 b_1)^2}}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Conditions of parallelism and perpendicularity of two lines :

Case I. When dc's of two lines AB and CD, say ℓ_1, m_1,n_1 and ℓ_2, m_2, n_2 are known

AB || CD $\Leftrightarrow \ell_1 = \ell_2, m_1 = m_2, n_1 = n_2$

AB ⊥ CD $\Leftrightarrow \ell_1 \ell_2 + m_1 m_2 + n_1 n_2 = 0$.

Case II. When dr's of two lines AB and CD, say : a_1, b_1, c_1 and a_2, b_2, c_2 are known

AB || CD $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

AB ⊥ CD $\Leftrightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.

Area of a triangle :

Let A(x_1, y_1, z_1); B(x_2, y_2, z_2) and C(x_3, y_3, z_3) are vertices of a triangle. Then

dr's of AB = $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$= a_1, b_1, c_1$ (say)

and AB = $\sqrt{a_1^2 + b_1^2 + c_1^2}$

dr's of BC = $x_3 - x_2, y_3 - y_2, z_3 - z_2$

$= a_2, b_2, c_2$(say)

and BC = $\sqrt{a_2^2 + b_2^2 + c_2^2}$

Now $\sin B = \frac{\sqrt{\Sigma(b_1 c_2 - b_2 c_1)^2}}{\sqrt{\Sigma a_1^2}\sqrt{\Sigma a_2^2}}$

$= \frac{\sqrt{\Sigma(b_1 c_2 - b_2 c_1)^2}}{AB.BC}$

$\therefore$ Area of ΔABC $= \frac{1}{2}$ AB. BC sin B

$= \frac{1}{2}\sqrt{\Sigma(b_1 c_2 - b_2 c_1)^2}$

Projection of a line segment joining two points on a line :

Let PQ be a line segment where P ≡ (x_1, y_1, z_1) and Q ≡ (x_2, y_2, z_2); and AB be a given line with dc's as l, m, n. If P'Q' be the projection of PQ on AB, then

P'Q' = PQ cos θ

where θ is the angle between PQ and AB. On replacing the value of cos θ in this, we shall get the following value of P'Q'.

P'Q' = $l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

Projection of PQ on x-axis : $a = |x_2 - x_1|$

Projection of PQ on y-axis : $b = |y_2 - y_1|$

Projection of PQ on z-axis : $c = |z_2 - z_1|$

Length of line segment PQ = $\sqrt{a^2 + b^2 + c^2}$

* If the given lines are $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and $\frac{x-\alpha'}{\ell'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$, then condition for intersection is

- If the given lines are $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and $\frac{x-\alpha'}{\ell'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$, then condition for intersections is

$$\begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ \ell & m & n \\ \ell & m' & n' \end{vmatrix} = 0$$

Plane containing the above two lines is

$$\begin{vmatrix} x-\alpha & y-\beta & z-\gamma \\ \ell & m & n \\ \ell' & m' & n' \end{vmatrix} = 0$$

Condition of coplanarity if both the lines are in general form:

Let the lines be

$ax + by + cz + d = 0 = a'x + b'y + c'z + d'$

and $\alpha x + \beta y + \gamma z + \delta = 0 = \alpha' x + \beta' y + \gamma' z + \delta'$

These are coplanar if $\begin{vmatrix} a & b & c & d \\ a' & b' & c' & d' \\ \alpha & \beta & \gamma & \delta \\ \alpha' & \beta' & \gamma' & \delta' \end{vmatrix} = 0$

Reduction of non-symmetrical form to symmetrical form:

Let equation of the line in non-symmetrical form be'

$a_1x + b_1y + c_1z + d_1 = 0;\ a_2x + b_2y + c_2z + d_2 = 0.$

To find the equation of the line in symmetrical form, we must know (i) its direction ratios (ii) coordinates of any point on it.

- **Direction ratios :** Let ℓ, m, n be the direction ratios of the line. Since the line lies in both the planes, it must be perpendicular to normals of both planes. So

$a_1\ell + b_1m + c_1n = 0;\ a_2\ell + b_2m + c_2n = 0$

From these equations, proportional values of ℓ, m, n can be found by cross-multiplication as

$$\frac{\ell}{b_1c_2 - b_2c_1} = \frac{m}{c_1a_2 - c_2a_1} = \frac{n}{a_1b_2 - a_2b_1}$$

- **Point on the line :** Note that as ℓ, m, n cannot be zero simultaneously, so at least one must be non-zero. Let $a_1b_2 - a_2b_1 \neq 0$, then the line cannot be parallel to *xy*-plane, so it intersect it. Let it intersect *xy*-plane in $(x_1, y_1, 0)$. Then

$a_1x_1 + b_1y_1 + d_1 = 0$ and $a_2x_1 + b_2y_1 + d_2 = 0$

Solving these, we get a point on the line. Then its equation becomes

$$\frac{x-x_1}{b_1c_2 - b_2c_1} = \frac{y-y_1}{c_1a_2 - c_2a_1} = \frac{z-0}{a_1b_2 - a_2b_1}$$

or $$\frac{x - \frac{b_1d_2 - b_2d_1}{a_1b_2 - a_2b_1}}{b_1c_2 - b_2c_1} = \frac{y - \frac{d_1a_2 - d_2a_1}{a_1b_2 - a_2b_1}}{c_1a_2 - c_2a_1} = \frac{z-0}{a_1b_2 - a_2b_1}$$

Note : If $\ell \neq 0$, take a point on yz –plane as $(0, t_1, z_1)$ and if $m \neq 0$, take a point on xz-plane as $(x_1, 0, z_1)$

- **Skew lines :** The straight lines which are not parallel and non-coplanar i.e. non-intersecting are called skew lines.

If $\Delta = \begin{vmatrix} x-\alpha & y-\beta & z-\gamma \\ \ell & m & n \\ \ell' & m' & n' \end{vmatrix} \neq 0$, the lines are skew.

Shortest distance : Suppose the equation of the lines are $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$

and $\frac{x-\alpha'}{\ell'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$. Then

$$\text{S.D.} = \frac{(\alpha-\alpha')(mn'-m'n) + (\beta-\beta')(n\ell'-n'\ell)(\ell m'-\ell' m)}{\sqrt{\Sigma(mn'-m'n)^2}}$$

$$= \begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ \ell & m & n \\ \ell' & m' & n' \end{vmatrix}$$

Some results for plane and straight line:

(i) General equation of a plane :

$ax + by + cz + d = 0$

where a, b, c are dr's of a normal to this plane.

(ii) Equation of a straight line :

General form : $\left.\begin{matrix} a_1x + b_1y + c_1z + d_1 = 0 \\ a_2x + b_2y + c_2z + d_2 = 0 \end{matrix}\right\}$

(In fact it is the straight line which is the intersection of two given planes)

Symmetric form : $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

where (x_1, y_1, z_1) is a point on this line and a, b, c are its dr's

(iii) Angle between two planes :

If θ be the angle between planes $a_1x + b_1y\ \ c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, then

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

(In fact angle between two planes is the angle between their normals.)

Further above two planes are

parallel $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

perpendicular $\Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

MATHS

PROGRESSION & MATHEMATICAL INDUCTION

Mathematics Fundamentals

Arithmetic Progression (AP)

AP is a progression in which the difference between any two consecutive terms is constant. This constant difference is called **common difference** (c.d.) and generally it is denoted by d.

Standard form: Its standard form is

$a + (a + d) + (a + 2d) + ...$

General term :

$T_n = a + (n - 1)\, d$

If $T_n = l$ then it should be noted that

(i) $d = \dfrac{\ell - a}{n-1}$ (ii) $n = 1 + \dfrac{\ell - a}{d}$

Note: a, b, c are in AP $\Leftrightarrow 2b = a + c$

Sum of n terms of an AP :

$$S_n = \frac{n}{2}(a + \ell)$$

where l is last term (nth term). Replacing the value of l, it takes the form

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Arithmetic Mean :

(i) If A be the AM between two numbers a and b, then $A = \dfrac{1}{2}(a + b)$

(ii) The AM of n numbers $a_1, a_2, ..., a_n$

$= \dfrac{1}{n}\,(a_1 + a_2 + ... + a_n)$

(iii) n AM's between two numbers

If $A_1, A_2, ..., A_n$ be n AM's between a and b then a $A_1, A_2, ..., A_n$, b is an AP of $(n + 2)$ terms. Its common difference d is given by

$T_{n+2} = b = a + (n + 1)d \Rightarrow d = \dfrac{b-a}{n+1}$

so $A_1 = a + d$, $A_2 = a + 2d$, ..., $A_n = a + nd$.

Sum of n AM's between a and b

$\therefore\ \Sigma A_n = n(A)$

Assuming numbers in AP :

(i) When number of terms be odd

Three terms : $a - d, a, a + d$

Five terms : $a - 2d, a - d, a, a + d, a + 2d$

....

(ii) When number of terms be even

Four terms: $a - 3d, a - d, a + d, a + 3d$

Six terms : $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$

....

Geometrical Progression (GP) :

A progression is called a GP if the ratio of its each term to its previous term is always constant. This constant ratio is called its **common ratio** and it is generally denoted by r.

Standard form : Its standard form is

$a + ar + ar^2 + ...$

General term : $T_n = ar^{n-1}$

a, b, c are in GP $\Leftrightarrow \dfrac{b}{a} = \dfrac{c}{b} \Leftrightarrow b^2 = ac$

Sum of n terms of a GP :

The sum of n terms of a GP $a + ar + ar^2 + ...$ is given by

$$S_n = \begin{cases} \dfrac{a(1-r^n)}{1-r} = \dfrac{a - \ell r}{1-r}, \text{when } r < 1 \\ \dfrac{a(r^n - 1)}{r-1} = \dfrac{\ell r - a}{r-1}, \text{when } r > 1 \end{cases}$$

when $\ell = T_n$.

Sum of an infinite GP :

(i) When r > 1, then $r^n \to \infty$, so $S_n \to \infty$ Thus when $r > 1$, the sum S of infinite GP $= \infty$

(ii) When $|r| < 1$, then $r^n \to 0$, so

$S = \dfrac{a}{1-r}$

(iii) When $r = 1$, then each term is a so $S = \infty$.

Geometric Mean :

(i) If G be the GM between a and b then

$G = \sqrt{ab}$

(ii) G.M. of n numbers $a_1, a_2 ....., a_n = (a_1 a_2 a_3 .....a_n)^{1/n}$

(iii) n GM's between two numbers

$\Rightarrow r = (b/a)^{1/n+1}$

Product of n GM's between a and b

Product of GM's = $(ab)^{n/2} = G^n$

Assuming numbers in GP :

(i) When number of terms be odd

Three terms : $a/r, a, ar$

Five terms : $a/r^2, a/r, a, ar, ar^2$

....

(ii) When number of terms be even

Four terms : $a/r^3, a/r, ar, ar^3$

Six terms : $a/r^5, a/r^3, a/r, ar, ar^3, ar^5$

Arithmetic-Geometric Progression :

If each term of a progression is the product of the corresponding terms of an AP and a GP, then it is called arithmetic-geometric progression (AGP). For example:

$a, (a + d)r, (a + 2d)r^2$ ....

$T_n = [a + (n - 1)d]\, r^{n-1}$

$$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$$

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} \qquad |r| < 1$$

Harmonic Progression :

A progression is called a harmonic progression (HP) if the reciprocals of its terms are in AP.

Standard form : $\frac{1}{a} + \frac{1}{a+d} + \frac{1}{a+2d} + \ldots\ldots\ldots\ldots$

General term : $T_n = \frac{1}{a+(n-1)d}$

$\therefore$ a, b, c are in HP $\Leftrightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Leftrightarrow b = \frac{2ac}{a+c}$

Harmonic Mean :

(i) If H be a HM between two numbers a and b, then

$$H = \frac{2ab}{a+b} \text{ or } \frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

(ii) To find n HM's between a and b we first find n AM's between 1/a and 1/b, then their reciprocals will be the required HM's.

Relations between AM, GM and HM :

$G^2 = AH$

$A > G > H$, when $a, b > 0$.

If A and AM and GM respectively between two positive numbers, then those numbers are

$$A + \sqrt{A^2 - G^2}, A - \sqrt{A^2 - G^2}$$

Some Important Results :

- If number of terms in an AP/GP/HP is odd then its mid term is the AM/GM/HM between the first and last term.
- If number of terms in an AP/GP/HP is even the AM/GM/HM of its two middle terms is equal to the AM/GM/HM between the first and last term.
- a, b, c are in AP, GP and HP $\Leftrightarrow a = b = c$
- a, b, c are in AP and HP $\Rightarrow a, b, c$ are in GP.
- a, b, c are in AP

$\Leftrightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP. $\Leftrightarrow bc, ca, ab$ are in HP.

- a, b, c are in GP $\Leftrightarrow a^2, b^2, c^2$ are in GP.
- a, b, c are in GP $\Leftrightarrow$ loga, logb, logc are in AP.
- a, b, c are in GP $\Leftrightarrow \log_a m\ \log_b m, \log_c m$ are in HP.
- $a, b, c\ d$ are in GP $\Leftrightarrow a + b, b + c, c + d$ are in GP.
- a, b, c are in AP $\Leftrightarrow \alpha^a, \alpha^b, \alpha^c$ are in GP ($\alpha \in R_0$)

Principle of Mathematical Induction :

It states that any statement P(n) is true for all positive integral values of n if

(i) P(1) is true i.e., it is true for $n = 1$.

(ii) P(m) is true $\Rightarrow P(m + 1)$ is also true

i.e., if the statement is true for $n = m$ then it must also be true for $n = m + 1$.

Some Formula based on the Principle of Induction :

- $\Sigma n = 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

(Sum of first n natural numbers)

- $\Sigma(2n - 1) = 1 + 3 + 5 + \ldots + (2n - 1) = n^2$

(Sum of first n odd numbers)

- $\Sigma 2n = 2 + 4 + 6 + \ldots\ldots + 2n = n(n + 1)$

(Sum of first n even numbers)

- $\Sigma n^2 = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)\ (2n+1)}{6}$

(Sum of the squares of first n natural numbers)

- $\Sigma n^3 = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{n^2(n+1)^2}{4}$

(Sum of the cubes of first n natural numbers)

Application in Solving Objective Question :

For solving objective question related to natural numbers we find out the correct alternative by negative examination of this principle. If the given statement is P(n), then by putting n = 1, 2, 3, ... in P(n), we decide the correct answer.

We also use the above formulae established by this principle to find the sum of n terms of a given series. For this we first express T_n as a polynomial in n and then for finding S_n, we put Σ before each term of this polynomial and then use above results of Σn, Σn^2, Σn^3 etc.

Based on New Pattern

IIT-JEE 2012

XtraEdge Test Series # 3

Time : 3 Hours

Syllabus : Physics : Calorimetry, K.T.G., Thermodynamics, Heat Transfer, Thermal expansion, Transverse wave, Sound wave, Doppler's effect, Atomic Structure, Radioactivity, X-ray, Nuclear Physics, Matter Waves, Photoelectric Effect, Practical Physics. **Chemistry :** Chemical Equilibrium, Acid Base, Ionic Equilibrium, Classification & Nomenclature, Isomerism , Hydrogen Family, Boron Family & Carbon Family, S-block elements, Nitrogen Family, Oxygen Family, Halogen Family & Noble Gas, Salt Analysis, Metallurgy, Co-ordination Compounds, Transitional Elements. **Mathematics:** Point, Straight line, Circle, Parabola, Ellipse, Hyperbola, Vector, 3-D, Probability, Determinants, Matrices.

Instructions : [Each subject contain]

Section – I : Question 1 to 8 are multiple choice questions with only one correct answer. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer.

Section – II : Question 9 to 13 are multiple choice questions with multiple correct answer. ***+3 marks*** will be awarded for correct answer and ***No Negative marks*** for wrong answer.

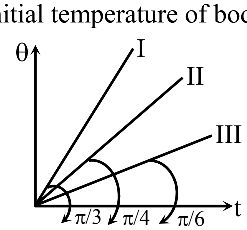
Section – III : Question 14 to 18 are passage based single correct type questions. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer

Section – IV : Question 19 to 27 are Numerical Response Question (single digit Ans. type) ***+4 marks*** will be awarded for correct answer and ***No Negative marks*** for wrong answer.

PHYSICS

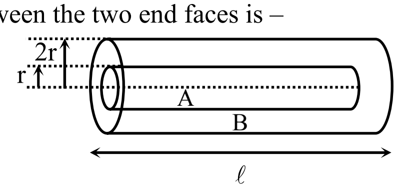
Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. Three bodies A, B and C of masses m, m and $\sqrt{3}$ m respectively are supplied heat at a constant rate. The change in temperature θ versus time t graph for A, B and C are shown by I, II and III respectively. If their specific heat capacities are S_A, S_B and S_C respectively then which of the following relation is correct? (Initial temperature of body is 0ºC) -



(A) $S_A > S_B > S_C$ (B) $S_B = S_C < S_A$
(C) $S_A = S_B = S_C$ (D) $S_B = S_C > S_A$

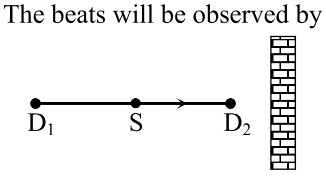
2. A composite cylinder is made of two different materials A and B of thermal conductivities K_A and K_B. The dimensions of the cylinder are as shown in the figure. The thermal resistance of the cylinder between the two end faces is –



(A) $\frac{\ell}{\pi r^2(K_A + 3K_B)}$ (B) $\frac{\ell}{\pi r^2}\left(\frac{1}{K_A} + \frac{3}{K_B}\right)$

(C) $\frac{\ell}{\pi r^2}\left(\frac{1}{K_A} + \frac{4}{K_B}\right)$ (D) $\frac{\ell}{\pi r^2(K_A + 4K_B)}$

3. Two stationary detector D_1 and D_2 and a moving source of sound are arranged as shown in the figure. The beats will be observed by –



(A) both D_1 and D_2 (B) only D_1
(C) only D_3 (D) none of these

4. The intensity level of a point isotropic source is 40 dB at a distance of 5 m from the source. The minimum distance at which the sound source will be inaudible is -
(A) 50 m (B) 500 m
(C) 100 m (D) infinite

5. The radius of the shortest orbit in a one-electron system is 18 pm. It may be -
(A) hydrogen (B) deuterium
(C) He^+ (D) Li^{++}

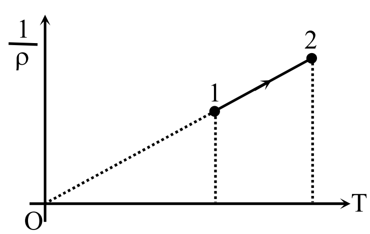
6. An electron with kinetic energy E eV collides with a hydrogen atom in the ground state. The collision will be elastic -
(A) for all values of E
(B) for E < 10.2 eV
(C) for E < 13.6 eV
(D) only for E < 3.4 eV

7. The absorption coefficient of x-rays for a given wavelength is larger for -
(A) lithium (B) copper
(C) aluminium (D) lead

8. When a photon falls on a metal, it is absorbed through a distance before ejection of photoelectron. This distance is given by -
(A) 1Å (B) 10^4 Å (C) 100Å (D) 10^8 Å

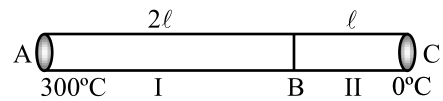
Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. The adjoining figure shows the graph of $\frac{1}{\rho}$ vs T for an ideal gas undergoing a reversible process. Here ρ represents the density, while T represents the temperature 0 –



(A) the process represents isobaric compression
(B) the internal energy of the gas decreases
(C) work is done by the gas on the surroundings
(D) heat is absorbed by the gas during the process.

10. Two conducting rods of the same cross-section are connected end to end, while the temperature at A and C are maintained at θ_A = 300ºC and θ_C = 0ºC, respectively. There is no loss of heat from the sides of the rods. Let θ_B be the temperature of function B –



(A) if θ_B = 200ºC in the steady state, the conductivities of the rods are equal
(B) if θ_B < 50ºC in the steady state, $K_I < K_{II}$
(C) if θ_B > 200ºC in the steady state, $K_I > K_{II}$
(D) if θ_B = 100ºC in the steady state, $K_I < K_{II}$
Here K_I, K_{II} denote the thermal conductivities of the rods I and II respectively.

11. Select the correct statement -
(A) The waves created in metal may be transverse waves
(B) The waves created in metal may be longitudinal waves
(C) The waves created in gas must be transverse mechanical waves
(D) The waves created in gas may be transverse mechanical waves

12. The maximum kinetic energy of photon electrons ejected from a photometer when it is irradiated with radiation of wavelength 400nm is 1 eV. If the threshold energy of surface is 1.9 eV -
(A) the maximum K.E. of photoelectrons when it is irradiated with 500 nm photon will be 0.42 eV
(B) the maximum K.E. in case (A) will be 1.725eV
(C) the longest wavelength which will eject photoelectrons is nearly 610 nm
(D) All of the above

13 The shortest wavelength produced in an x-ray tube operated at 0.5 million volt is -
(A) depends on the target material
(B) 0.025 Å approximately
(C) double of the shortest wavelength produced with the tube operating at 1 million volt
(D) depends only on the target material and not on the operating voltage

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage-II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage # 1 (Ques. 14 to 15)

In successive radioactive transformation a radioactive nuclei decays to another nuclei which in turn decays to another stable nuclei, as shown below

$$A \xrightarrow[\lambda_1]{} B \xrightarrow[\lambda_2]{} C \text{ (stable)}$$

λ_1 & λ_2 are the decay constants for the transformations respectively. Initially number of nuclei of A that are present are N_0 and at any instant 't' number of nuclei of B are

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\ e^{-\lambda_1 t}\ [1 - e^{-(\lambda_2-\lambda_1)t}]$$

14. The number of nuclei of C at any instant 't' is -

(A) $N_3 = N_0\,(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1}\ e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1}\ e^{-\lambda_1 t})$

(B) $N_3 = N_0(1 - \frac{\lambda_1}{\lambda_2 - \lambda_1}\ e^{-\lambda_2 t} + \frac{\lambda_2}{\lambda_2 - \lambda_1}\ e^{-\lambda_1 t}$ a)

(C) $N_3 = N_0(1 - \frac{\lambda_1}{\lambda_2 - \lambda_1}\ e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1}\ e^{-\lambda_1 t})$

(D) $N_3 = N_0(\frac{\lambda_1}{\lambda_2 - \lambda_1}\ e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1}\ e^{-\lambda_1 t})$

15. At radioactive equilibrium the rate of production of nuclei C is -

(A) $\frac{\lambda_2\lambda_1}{\lambda_2 + \lambda_1}\ N_0 e^{-\lambda_1 t}\ [1 - e^{-(\lambda_2+\lambda_1)t}]$

(B) $\frac{\lambda_2\lambda_1}{\lambda_2 - \lambda_1}\ N_0\, e^{-\lambda_1 t}\ [1 - e^{-(\lambda_2-\lambda_1)t}]$

(C) $\frac{\lambda_2\lambda_1}{\lambda_2 - \lambda_1}\ N_0\, e^{-\lambda_1 t}\ [1 + e^{-(\lambda_2-\lambda_1)t}]$

(D) $\frac{\lambda_2\lambda_1}{\lambda_2 - \lambda_1}\ N_0\, e^{-\lambda_2 t}\ [1 - e^{-(\lambda_2-\lambda_1)t}]$

Passage # 2 (Ques. 16 to 18)

Surface temperature of Sun has to be estimated for this most probable energy E for photons emitted by a black body of temperature T is found. Intensity is proportional to $E^5 e^{-E/KT}$ i.e., $I \propto E^5\ e^{-E/kT.}$

The Balmer lines of hydrogen span the visible frequency range. The human eye has evolved to be most sensitive to sunlight.

If n_E is the number of photons with energy E then distribution of n_E is given by

$$P(n_E) \sim \frac{I(E)}{E}$$

16. The most probable energy E for photons emitted by a black body of temperature T is -

(A) KT (B) 2KT

(C) 3KT (D) 4 KT

17. The maximum energy of the Balmer lines which fall in the visible range is -

(A) 1.9 eV (B) 3.4 eV

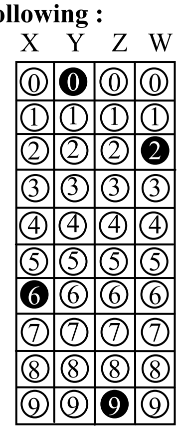
(C) 5.2 eV (D) 6.4 eV

18. If the human eye is sensitive to sunlight, the maximum surface temperature of sun is -

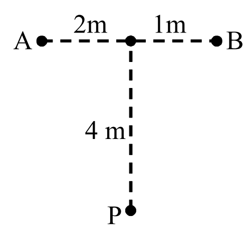
(A) 0.5×10^4 K (B) 2×10^4 K

(C) 3×10^4 K (D) 4×10^4 K

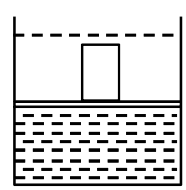
This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



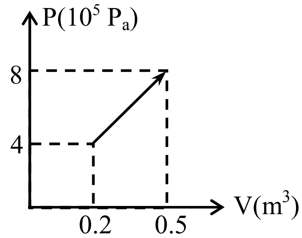
19. Two small sound sources A and B emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s, for what minimum frequency does destructive interference occur at point P. Answer is in the form of $n \times 10^2$ Hz. What is n ?



20. A cylinder contains 0.15 kg of hydrogen. The cylinder is closed by a piston supporting a weight of 74 kg (see fig.), $n \times 10^3$ J amount of heat is given to lift the weight by 1.2 m ? The process should be assumed isobaric, the heat capacity of the vessel and the external pressure should be neglected. Find n (n is single digit.)



21. Upon expansion the pressure of a gas rose linearly. Quantity of heat supplied for this is $x \cdot y \times 10^5$J, where x and y are single digit number find x. The gas is monotonic.



22. The photoelectric current in a vacuum photocell is reduced to zero when its cesium (ϕ = 1.89eV) electrode is irradiated by radiation of wavelength λ = 2700 Å and a decelerating voltage V = 3V is applied. Then the magnitude of outer contact potential difference is N × 10^{-1} V, then the value of N is –

23. The De-Broglie wavelength of electron in the third Bohr orbit of hydrogen in 10^{-9} m is (given radius of first Bohr orbit is 5.3 × 10^{-11} m) -

24. The binding energy of an electron in the ground state of He atom is equal to 24.6 eV. The energy required to remove both the electrons (if the ionisation energy of hydrogen is 13.6 eV) is N × 10^1 eV then N is equal to -

25. The nucleus ${}_{92}U^{238}$ is unstable against α-decay with a half-life of about 4.5 × 10^9 years. Then the kinetic energy of the emitted α-particle in MeV is [m (${}_{92}U^{238}$) = 238.05081u ; m(${}_{2}He^4$) = 4.00260 u; m(${}_{90}Th^{234}$) = 234.04363u]

26. A polonium (${}_{84}P_0{}^{209}$) nucleus transforms into one of lead (${}_{82}Pb^{207}$) by emitting an α-particle, then the kinetic energy of the α-particle in MeV is -
[m (P_0) = 209.98297u ; m (Pb) = 205.97446 m (α-particle) = 4.00260 u]

27. A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength 5.76 × 10^{-15} m. If the mass of daughter nucleus is 223.610 amu and that of α-particle is 4.002 amu. The mass of the parent nucleus is 22X amu then find X appearing in the number 22X.
(1 amu = 931.47 MeV/c^2)

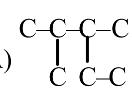
CHEMISTRY

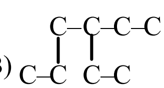
Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

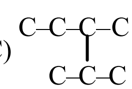
1. Which out of the following gases is obtained when ammonium dichromate is heated -
(A) Oxygen (B) Ammonia
(C) Nitrogen (D) Nitrous oxide

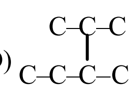
2. The solubility products of $Al(OH)_3$ and $Zn(OH)_2$ are 8.5 × 10^{-23} and 1.8 × 10^{-14} respectively. If NH_4OH is added to a solution containing Al^{3+} and Zn^{2+} ions, then substance precipitated first is -
(A) $Al(OH)_3$ (B) $Zn(OH)_2$
(C) Both together (D) None at all

3. Out of following carbon chains which one is different from other three chains –

(A) 

(B) 

(C) 

(D) 

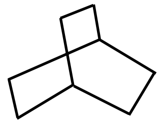
4. Removal of $FeWO_4$ is not possible by gravity separation because -
(A) $FeWO_4$ is having wetting characteristics
(B) $FeWO_4$ is having magnetic property
(C) $FeWO_4$ having density almost that of SnO_2
(D) None of these

5. $A \xrightarrow{D} B + C$ for this reaction it is given that

time	O	t	∞
vol of reagent	V_1	V_2	V_3

If the reagent reacts with B, C and D which is the correct expression for k
(A) $k = \frac{1}{t} \ell n \frac{V_3 - V_1}{V_2 - V_3}$ (B) $k = \frac{1}{t} \ell n \frac{V_1 - V_3}{V_3 - V_2}$
(C) $k = \frac{1}{t} \ell n \frac{V_3 - V_1}{V_3 - V_2}$ (D) $k = \frac{1}{t} \ell n \frac{V_3 - V_{1/2}}{V_3 - V_2}$

6. A colourless solid (A) on heating evolve CO_2 and give a white residue (B) soluble in water. (B) also give CO_2 when treated with dilute acid. (A) is -
(A) Na_2CO_3 (B) $NaHCO_3$
(C) $CaCO_3$ (D) $Ca(HCO_3)_2$

7. The name of the compound  is -

(A) bicyclo [2.2.1] octane
(B) bicyclo [1.1.1] octane
(C) 1, 4-bismethylenecyclohexane
(D) bicyclo [2.2.2] octane

8. Atomic radii of alkali metals (M) follow the order : Li < Na < K < Rb but ionic radii in aqueous solution follow the reverse order $Li^+ > Na^+ > K^+ > Rb^+$. The reason for the reversed order is -

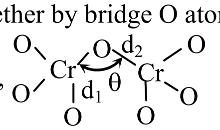
(A) Increase in the ionisation energy
(B) Decrease in the metallic bond character
(C) Increase in the electropositive character
(D) Decrease in the amount of hydration

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. When KI solution is added dropwise to the Bi^{3+} solution a black ppt of (X) is obtained. On adding excess of KI solution orange coloured solution (Y) is obtained. On addition of water again black ppt is reappeared. The (Y) in water on heating it turns orange ppt (Z). Then X, Y, and Z are-

(A) (X) ≡ BiI_3 (B) (Y) ≡ $[BiI_4]^-$
(C) (Z) ≡ BiOI (D) (Y) ≡ BiI_6^-

10. For the CrO_4^{2-} and $Cr_2O_7^{2-}$ which of the following is/are correct ?

(A) In $Cr_2O_7^{2-}$, two tetrahedral CrO_4^{2-} units are joined together by bridge O atom
(B) In $Cr_2O_7^{2-}$, 

$\theta > 109.5°$; $d_1 < d_2$
(C) The reduction potential of $Cr_2O_7^{2-}$ in acidic medium is greater than the reduction potential of CrO_4^{2-} in the acidic medium
(D) $[CrO_8]^{3-}$ (peroxochromate) is peramagnetic in nature

11. Which of the following is/are correct ?

(A) The efficiency of a solid catalyst depends upon its surface area
(B) Catalyst operates by providing alternate path for the reaction that involves lower activation energy
(C) Catalyst lowers the activation energy of forward reaction only without affecting the activation energy of backward reaction
(D) Catalyst does not affect the overall enthalpy change of the reaction

12. Which of the following statement is/are correct?

(A) $[Ni(CO)_4]$ is tetrahedral, paramagnetic, sp^3 hybridised
(B) $[Ni(CN)_4]^{2-}$ is square planar, diamagnetic, dsp^2 hybridised
(C) $[Ni(CO)_4]$ is tetrahedral, diamagnetic sp^3 hybridised
(D) $[NiCl_4]^{2-}$ is tetrahedral, paramagnetic, sp^3 hybridised

13. Which of the following hydrides are covalent and polymeric ?

(A) Ca (B) Ba
(C) Be (D) Mg

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage- II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage # 1 (Ques. 14 & 15)

pH scale was designed by Sorensen to represent $[H^+]$ concentrations is a suitable manner. For a given solution,
$pH = -\log[H^+]$
$pOH = -\log[OH^-]$
and $pH + pOH = pK_w$
where $pK_w = -\log K_w$ (K_w = Ionic product of water)
Nature of salt solutions and their resulting pH are determined by comparing the strength of acid & base used for salt formation.

Acid	Base	Nature
strong	strong	neutral ($pH = 1/2\ pK_w$)
strong	weak	acidic ($pH < 1/2\ pK_w$)
weak	strong	basic ($pH > 1/2\ pK_w$)
weak	weak	unpredictable

14. pH of $NaNO_3$ solution at t°C was found to be 6. find the pH of KCl solution at a temperature (t – 65°C)

(A) 7 (B) 8
(C) 6 (D) None of these

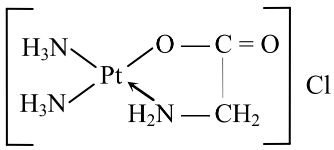
15. Nature of $AlCl_3$ solution is …….. while …….. solution is basic in nature.

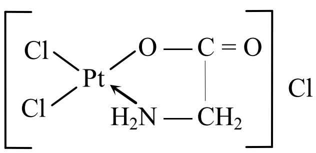
(A) Acidic, NaCl (B) Acidic, CH_3COONa
(C) Basic, CH_3COONa (D) Basic, NaCl

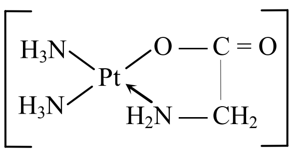
Passage # 2 (Ques. 16 to 18)

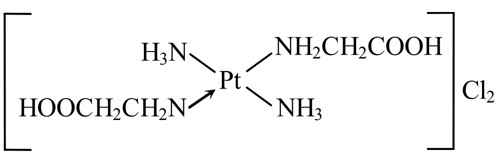
Chemically cis and trans isomers are separated by Grinberg's method. In this method cis and trans isomer reacts separately with the ligands (L) containing two donor atoms separated by two to four other atoms. When the ligands (L) attacks cis positions then five or six member ring is produced. With trans isomer L acts as monodentate ligand.

16. With cis-$[Pt(NH_3)_2Cl_2]$ when H_2NCH_2COOH reacts the product is (X). The (X) is –

(A) 

(B) 

(C) 

(D) 

17. With cis-$[Pt(NH_3)_2Cl_2]$ reacts at the faster rate with NH_2CH_2COOH than that of trans - $[Pt(NH_3)_2Cl_2]$ because -

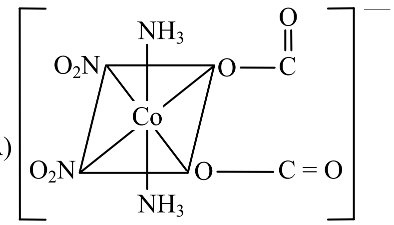
(A) With cis isomer of $Pt(NH_3)_2Cl_2$, NH_2CH_2COOH produces five member cyclic ring which is stable

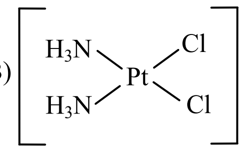
(B) With cis-$[Pt(NH_3)_2Cl_2]$ when NH_2CH_2COOH react the entropy change of the reaction is positive

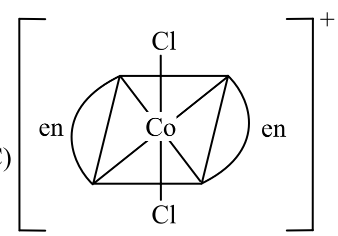
(C) With cis -isomer when H_2NCH_2COOH react there is vicinity of the attacking nucleophile with the incipient leaving group but with trans-isomer when H_2NCH_2COOH react there is steric crowding.

(D) All of the above are correct

18. Which of the following species are optically inactive ?

(A) 

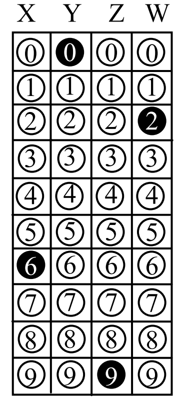
(B) 

(C) 

(D) All of the given species are optically inactive

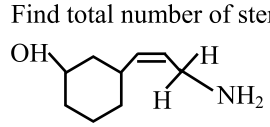
This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :

X Y Z W



19. In molecule of nitro glycerin the number of N atoms present are …………

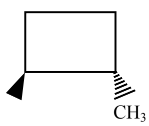
20. Find total number of stereocentre.

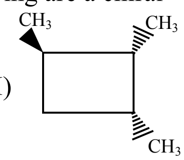


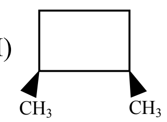
21. Calculate pH of a buffer solution prepared by dissolving 16.8 gram Na_2CO_3 in 500 ml of an aqueous solution containing 5.475 gram HCl.
(Ka for HCO_3^- = 5.63×10^{-11}; log 5.63 = 0.75 ; log 0.0566 = – 1.25).

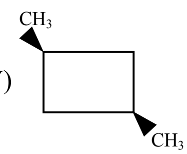
22. For Cu^{2+}, the spin only magnetic moment, $\mu = \sqrt{n_1}$ the value of n_1 is ....

23. How many no. of the following are a chiral

(I) 

(II) 

(III) 

(IV) 

24. Borazene, $B_3N_3H_6$, is isoelectronic and iso structural with benzene molecules, how many isotopic disubstituted borazene molecules, $B_3N_3H_4X_2$, are possible without changing the fundamental ring structure ?

25. In molecule of nitro glycerin the number of N atoms present are ....

26. Benitoite is represented as BaTi $[Si_3O_n]$, the value of n is ....

27. In can acid buffer solution, [HA] = 0.01 (M) and [NaA] = 0.1 (M) and for HA, $K_a = 10^{-5}$. In the given buffer solution the degree of hydrolysis, and $h \times 10^7$ is ....

MATHEMATICS

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1. Let ABCD is a convex quadrilateral in which $\angle BAC = 50°$, $\angle CAD = 60°$, $\angle CBD = 30°$ & $\angle BDC = 25°$. If E is the point of intersection of AC & BD then $\angle AEB$ equals -
(A) 65º (B) 75º (C) 85º (D) 95º

2. p_1, p_2 are lengths of perpendicular from foci on tangent to ellipse and p_3, p_4 are perpendiculars from extremities of major axis and p from centre of ellipse on same tangent, then $\dfrac{p_1p_2 - p^2}{p_3p_4 - p^2}$ equals
(A) e (B) $\sqrt{e}$
(C) e^2 (D) None of these

3. Let $f(x) = \begin{vmatrix} x^2+1 & x-1 & x-2 \\ 2x & x-3 & x+4 \\ 3x-1 & 4x & 8x-2 \end{vmatrix} - ax^4 - bx^3 - cx^2 - dx - e$ and $f(x) = 0$ for all x, then equation $(a + 4e - 3c)x^2 + (b - 2e - a)x + 3c - 2e - b = 0$ has a root equal to -
(A) $\dfrac{e}{4a}$ (B) $\dfrac{4b}{e}$ (C) $\dfrac{4a}{b}$ (D) $\dfrac{b}{4a}$

4. If $\vec{a} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ $\forall\, x_1, x_2, x_3 \in \{1, 2, 3, ... 10\}$, then number of vectors $\vec{a}$ such that $\vec{a}\,.\,\vec{b} = 0$ is -
(A) 20 (B) 50
(C) 55 (D) 40

5. If A, B & C are matrices of order 2 such that $|A| = \dfrac{1}{4}$, $|B| = 9$, $|C| = 2$, then $|(3AC)(2B)^{-1}|$ is equal to -
(A) $\dfrac{1}{8}$ (B) $\dfrac{3}{4}$
(C) 2 (D) 6

6. A three digit number is selected at random from the set of all three digit numbers. The probability that the number selected has atleast two digits same is -
(A) $\dfrac{24}{25}$ (B) $\dfrac{9}{25}$ (C) $\dfrac{18}{25}$ (D) $\dfrac{7}{25}$

7. If $0 \le x \le 4$ and $0 \le y \le 4$ then probability of a point randomly selected from given intervals to lie inside or on the parabola $y^2 = x$ is -
(A) $\dfrac{3}{16}$ (B) $\dfrac{5}{16}$
(C) $\dfrac{2}{3}$ (D) $\dfrac{1}{3}$

8. If $\sum_{n=1}^{n} \alpha_n = an^2 + bn$, where a, b are constants and $\alpha_1, \alpha_2, \alpha_3 \in \{1, 2, 3........9\}$ and $25\alpha_1$, $37\alpha_2$, $49\alpha_3$ be three digit number then $\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ 5 & 7 & 9 \\ 25\alpha_1 & 37\alpha_2 & 49\alpha_3 \end{bmatrix}$ is equal to -

(A) $\alpha_1 + \alpha_2 + \alpha_3$ (B) $\alpha_1 - \alpha_2 + \alpha_3$
(C) 7 (D) 0

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. Two circles C_1 and C_2 intersect at two distinct points P & Q in a plane. Let a line passing through P meets circle C_1 & C_2 in A and B respectively. Let Y is mid point of AB and QY meets circle C_1 and C_2 in X and Z respectively, then-

(A) Y is mid point of XZ

(B) $\frac{XY}{YZ} = \frac{3}{1}$

(C) XY = YZ

(D) XY + YZ = 2YZ

10. If acute angle between the two asymptotes of hyperbola is $\frac{\pi}{3}$, then eccentricity of hyperbola is

(A) $\frac{2}{\sqrt{3}}$ (B) 2 (C) $\frac{4}{\sqrt{3}}$ (D) 4

11. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non coplanar vectors such that $\vec{r_1} = \vec{a} - \vec{b} + \vec{c}$, $r_2 = \vec{b} + \vec{c} - \vec{a}$, $r_3 = \vec{c} + \vec{a} + \vec{b}$ and $\vec{r} = 2\vec{a} + 4\vec{b} + 6\vec{c}$. If $\vec{r} = \lambda_1 \vec{r_1} + \lambda_2 \vec{r_2} + \lambda_3 \vec{r_3}$. Then which of the following is/are true -

(A) $\lambda_1, \lambda_2, \lambda_3$ are in A.P.

(B) $\lambda_1, \lambda_2, \lambda_3$ are in G.P.

(C) $\lambda_1 + \lambda_2 + \lambda_3 = 6$

(D) $\lambda_1, \lambda_2, \lambda_3$ are roots of the equation
$x^3 - 6x^2 + 11x - 6 = 0$

12. If $A(\alpha) = \begin{bmatrix} \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha \end{bmatrix}$, $\alpha \neq 0$ then -

(A) $2A(1) = A^2(1)$ (B) $A^3(1) = 9A(1)$
(C) adj.A does not exist (D) A^{-1} does not exist

13. Three numbers are selected at random from the set {1, 2, 3...........N}, one by one without replacement. If the first number is known to be smaller than second, then the probability that third selected number lies between the first two numbers is -

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{8}$

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Passage # 1 (Ques. 14 & 15)

Consider matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$,

$B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & -1 \\ 2 & 2 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 13 \\ 11 \\ 14 \end{bmatrix}$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that solutions of equations AX = C and BX = D represents two points $P(x_1, y_1, z_1)$ & $Q(x_2, y_2, z_2)$ respectively in 3D space. Let plane P′ is $x + y + z = 9$.

14. If RS is the reflection of line PQ in the plane P′, then the point which does not lie on RS is -

(A) (1, 5, 6) (B) (5, 3, 4)
(C) (7, 2, 3) (D) (3, 4, 2)

15. The value of det (adj (adjA)) is equal to -

(A) $(16)^4$ (B) $(16)^2$
(C) $-(16)^3$ (D) -16

Passage # 2 (Ques. 16 to 18)

a_1, a_2, a_3 are three consecutive terms of an increasing A.P., where a_1 and a_2 are prime numbers such that their sum is minimum possible odd prime number.

Urn 1 : Contains a_1 red and a_3 green balls
Urn 2 : Contains a_2 red and a_2 green balls
Urn 3 : Contains a_3 red a_1 green balls
P(i) Represents the probability of choosing i^{th} urn & P(R) represents probability of choosing red ball & similarly P(G) represents the probability of choosing green ball.

16. If P(i) α i^2 and one ball is drawn from one of these urns then -

(A) $P(G) = \frac{3}{7}$ (B) $P(R) = \frac{6}{7}$

(C) P(R) > P(G) (D) P(R) = P(G)

17. If P(i) = $\frac{1}{3}$, $\forall$ i = 1, 2, 3 and 2 balls are drawn randomly from one of these urns then the chance of drawing ball of different colours is -

(A) $\frac{3}{5}$ (B) $\frac{5}{9}$ (C) $\frac{2}{3}$ (D) $\frac{1}{5}$

18. If P(i) = $\frac{1}{3}$ $\forall$ i = 1, 2, 3 and an urn is chosen and balls are drawn one by one with replacement 10 times then probability that all drawings result in red balls is -

(A) $\frac{2^{10}+3^{10}+4^{10}}{6^{10}}$ (B) $\frac{4.2^{10}+3^{10}}{6^{10}}$

(C) $\frac{2^{10}+3^{10}}{6^{10}}$ (D) $\frac{2^{10}+3^{10}+4^{10}}{3.6^{10}}$

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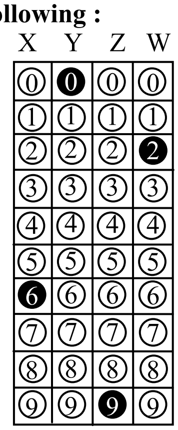


19. Two circles of radii r_1 & r_2 both touch coordinate axes & intersect orthogonally. Then value of $\frac{r_1}{r_2}$ (where $r_1 > r_2$) is $k + \sqrt{k^2-1}$ where k equals ($k \in$ I).

20. Real no. (x, y) satisfy the circle with unit radius & centered at origin. If maximum & minimum value of expression $\frac{4-y}{7-x}$ are M & m respectively then value of 2M + 6m is.

21. If M is foot of perpendicular from point P on a parabola to its directrix & SPM is an equilateral triangle where S is focus then SP is equal to λa, where λ equals

22. The product of perpendicular drawn from any point on $\frac{x^2}{9} - \frac{y^2}{16} = 1$ upon its asymptote is $\left(\frac{12}{5}\right)^k$ where k equals

23. Let $\vec{a}$ = secα $\hat{i}$ + ℓn $(2\pi\alpha - \alpha^2)$ $\hat{j}$ + $\sqrt{\cos\alpha}$ $\hat{k}$ and $\vec{b}$ = cos 5$\alpha$$\hat{i}$ + $\sqrt{\sec\alpha}$ $\hat{k}$. If $\vec{a}$ & $\vec{b}$ are perpendicular to each other then number of possible real solutions of α are.

24. If A = $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & y & z \end{bmatrix}$ is an orthogonal matrix then the value of x + y is equal to.

25. If $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix} = 2a^p\, b^q\, c^r$, then $p + q + r$ is equal to.

26. There are two possible values of A (say A_1 & A_2) in the solution of matrix equation

$$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$$

then find –27 ($A_1 + A_2$)

27. If equation of the plane through the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$ *ia* $ax - by + cz + 4 = 0$, then find the value of $\frac{10^3a + 10^2b + 10c}{342}$.

IIT-JEE 2013

XtraEdge Test Series # 3

Time : 3 Hours

Syllabus : Physics : Calorimetry, K.T.G.,Thermodynamics, Heat Transfer, Thermal expansion, Transverse wave, Sound wave, Doppler's effect. **Chemistry :** Chemical Equilibrium, Acid Base, Ionic Equilibrium, Classification & Nomenclature, Isomerism , Hydrogen Family, Boron Family & Carbon Family, S-block elements. **Mathematics:** Point, Straight line, Circle, Parabola, Ellipse, Hyperbola, Vector, 3-D.

Instructions : [Each subject contain]

Section – I : Question 1 to 8 are multiple choice questions with only one correct answer. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer.

Section – II : Question 9 to 13 are multiple choice questions with multiple correct answer. ***+3 marks*** will be awarded for correct answer and ***No Negative marks*** for wrong answer.

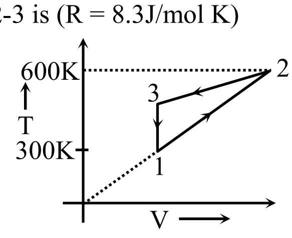
Section – III : Question 14 to 18 are passage based single correct type questions. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer

Section – IV : Question 19 to 27 are Numerical Response Question (single digit Ans. type) ***+4 marks*** will be awarded for correct answer and ***No Negative marks*** for wrong answer.

PHYSICS

Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. Two moles of an ideal gas are undergone a cyclic process 1 – 2 – 3 – 1. If net heat exchange in the process is – 300 J, then work done by the gas in the process 2-3 is (R = 8.3J/mol K)



(A) – 5000 J (B) 5000 J

(C) – 3000 J (D) None of these

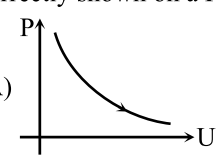
2. An equi-molar mixture of a monoatomic and a diatomic gas is subjected to a continuous and reversible process such that for an infinite small part of the process,

$$dQ = -\frac{1}{3}\,dU$$

where dQ is the heat input to the gas. The molar heat capacity of the two during the process is -

(A) $-\frac{1}{3}R$ (B) $\frac{2}{3}R$ (C) $-\frac{2}{3}R$ (D) $\frac{R}{3}$

3. The internal energy (U) and the density (ρ) of an ideal gas follow the law Uρ = constant during a particular process. The same process will be correctly shown on a P–U diagram as -

(A)


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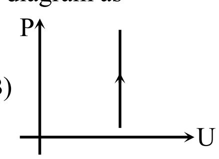
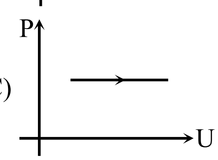
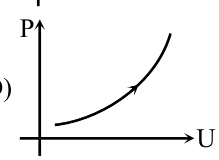
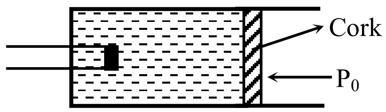


(C)


(D)


4. In the adjacent figure one mole of a monoatomic gas is enclosed in an adiabatic vessel by means of a tightly fitted cork. Heat is supplied to the gas at a constant rate Q = 4.5 × 10^{-2} J/s, by an electric heater. At time t = 0, temperature of the gas is 27ºC and its pressure is equal to atmospheric pressure P_0 = 10^5 Pa. If maximum frictional force offered to the cork by the walls of the vessel is 50 N, the time when cork will come out of the vessel is. (The cross-sectional area of the vessel A = 10^{-2} m^2. Take R = 8.3 J/mol K) –



(A) 41.5×10^2 s (B) 67.3×10^2 s
(C) 3.73×10^2 s (D) 37.3×10^2 s

5. There are two thin spheres A and B of the same material and same thickness. They emit like black bodies. Radius of A is double that of B. A and B of same temperature T. When A and B are kept in a room of temperature T_0 (< T), the ratio of their rates of cooling (rate of fall of temperature) is (assume negligible heat exchange between A and B) –

(A) 2 : 1 (B) 1 : 1 (C) 4 : 1 (D) 8 : 1

6. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is -

(A) 4 RT (B) 5 RT
(C) 15 RT (D) 11 RT

7. One end of a metal rod is dipped in boiling water and the other is dipped in melting ice.

(A) All parts of the rod are in thermal equilibrium with each other.
(B) We can assign a temperature to the rod.
(C) We can assign at temperature to the rod after steady state is reached
(D) The state of the rod does not change after steady state is reached

8. An anisotropic material has coefficient of linear expansion α, 2α and 3α along the three co-ordinate axis. Coefficient of cubical expansion of material will be equal to -

(A) 2α (B) $\sqrt[3]{6}\alpha$
(C) 6α (D) None of these

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. When the temperature of a copper coin is raised by 80ºC, its diameter increases by 0.2% -

(A) percentage rise in the area of a face is 0.4%
(B) percentage rise in the thickness is 0.4%
(C) percentage rise in the volume is 0.6%
(D) coefficient of linear expansion of copper is 0.25×10^{-4} / ºC

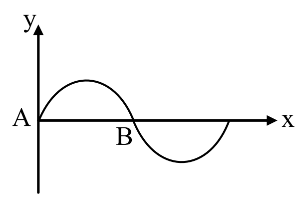
10. A vessel is partly filled with liquid. When the vessel is cooled to a lower temperature, the space in the vessel, unoccupied by the liquid remains constant. Then the volume of the liquid (V_L), volume of the vessel (V_V), the coefficient of cubical expansion of material of the vessel (γ_V) and of the liquid (γ_L) are related as -

(A) $\gamma_L > \gamma_V$ (B) $\gamma_L < \gamma_V$
(C) $\gamma_V/\gamma_L = V_V/V_L$ (D) $\gamma_V/\gamma_L = V_L/V_V$

11. With rise in temperature -

(A) Rubber contract
(B) A floating body sinks a little more
(C) Water contracts if temperature rises from 0ºC to 4ºC
(D) Water expands if temperature rises from 0ºC to 4ºC

12. The tension is a stretched string fixed at both ends is changed by 2%, the fundamental frequency is found to get changed by 15 Hz. Select the correct statement(s) –



(A) Wavelength of the string of fundamental frequency does not change
(B) Velocity of propagation of wave changes by 2%
(C) Velocity of propagation of wave changes by 1%
(D) Original frequency is 1500 Hz

13. The equation of a wave traveling on a string is given by y = 8 sin [5 m^{-1})x – (4 s^{-1})t]. Then-

(A) velocity of wave is 0.8 m/s
(B) the displacement of a particle of the string at t = 0 and x = $\frac{\pi}{30}$ m from the mean position is 4 m
(C) the displacement of a particle from the mean position at t = 0, x = $\frac{\pi}{30}$ m is 8 m
(D) velocity of the wave is 8 m/s

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage-II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage # 1 (Ques. 14 to 15)

A transverse pulse traveling in the positive x-direction without any change in shape is represented by the function y (x, t) which gives the displacement of the particle at the location x at a time t. It is given that

$y(x, 0) = \frac{1}{1+x^2}$ (at t = 0 s)

and $y(x, 2) = \frac{1}{x^2 - 2x + 2}$ (at t = 2s)

where y is measured in centimetre, x in metre and t in seconds.

The velocity of any particle on the waveform is given by

$v(x, t) = \frac{\partial y(x,t)}{\partial t}$,

the derivative of the displacement with respect to time t, regarding x as a constant (while taking the derivative)

The velocity of any particle on the waveform located at x is a function of x as well as the time t.

The mass density of the particle is 40 g/m along the x-axis.

14. The speed of the wave is -

(A) 1m/s (B) 2 m/s

(C) 0.5 m/s (D) none of the above

15. The maximum speed of any particle on the waveform equals -

(A) $\frac{\sqrt{3}}{4}$ cm/s (B) $\frac{\sqrt{3}}{16}$ cm/s

(C) $\frac{1}{16}$ cm/s (D) $\frac{3\sqrt{3}}{16}$ cm/s

Passage # 2 (Ques. 16 to 18)

A hot body placed in surroundings of temperature 10ºC obeys Newton's law of cooling :

$\frac{d\theta'}{dt} = -k\theta'$, where θ' is the excess temperature of the body above its surroundings and $k = \frac{7}{100}$ s^{-1}.

The thermal capacity of the body is 100 JK^{-1}.

(Take ln 2 = 0.7).

16. If the initial temperature of the body is 90ºC, after what time will the temperature of the body be equal to 50ºC ?

(A) 8.0 s (B) 10 s (C) 9.5 s (D) 10.5 s

17. The rate of loss of heat from the body, when its temperature is 70ºC, is -

(A) 320 W (B) 350W (C) 450W (D) 420W

18. Infra-red radiation is focused on the body, supplying it energy at the rate of 490W, 50% of which is absorbed. The equilibrium temperature of the body equals (Assume that Newton's law of cooling holds)

(A) 35ºC (B) 45ºC

(C) 80ºC (D) greater than 100ºC

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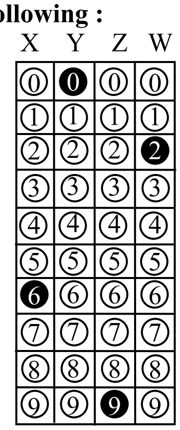


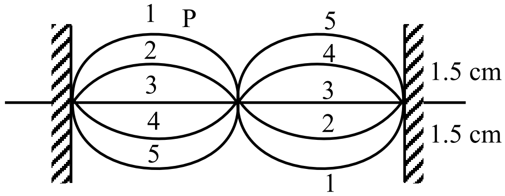
19. Two cylinders A and B filled with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is force to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is.....× 10^{-1}

20. A heat engine absorbs heat at 327 ºC and exhausts heat at 127 ºC. The maximum amount of work performed by the engine in joule per kilo calorie is-

21. 1 g of water on evaporation at atmospheric pressure forms 1671 cm^3 of steam. Heat of vaporization at this pressure is 540 cal/g. The increase in internal energy in K cal is...×10^1

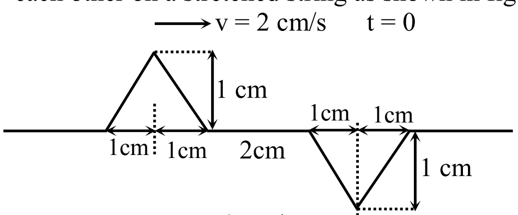
22. An observer at a distance of 800 m from a sound source heard first the sound signal which travelled through water and 1.785 later the signal which travelled through air. The velocity of sound in water is (x·y) × 10^3 m/s. Where x and y is the single digit non zero number, find x. The air temperature is 17ºC -

23. A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown is observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between. Speed of the traveling waves on the string is (x.y) m/s where x and y are single non zero digit number. Find x.



24. A long spring such as slinky is often used to demonstrate longitudinal waves. If mass of spring is m, length L and force constant K, then find the speed of longitudinal waves on the spring where m = 0.250 kg, L = 2.00 m K = 1.50 N/m.

25. Two triangular wave pulses are traveling towards each other on a stretched string as shown in figure.



Speed of each pulse is 2 cm/s. Find maximum displacement of particle of string at t = 1s. The leading edges of the pulses are 2.00 cm apart at t = 0.

26. A wave pulse described by the function

$$\gamma(x, t) = \frac{A^3}{A^2 + (x - vt)^2}$$

propagates down the string, where A = 1.00 cm, and v = 20.0 m/s. At the point x = 4.00 cm, at the t = n × 10^{-3}s the displacement is maximum. Find n.

27. Two trains whistles, A and B each have a frequency of 392 Hz. A is stationary and B is moving toward the right (away from A) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s. No wind is blowing. What is beat frequency detected by the listener.

CHEMISTRY

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1. Which of the following compound's prefix 'iso' is not correct –

(A) Iso pentane (B) Iso Hexane
(C) Iso butane (D) Iso octane

2. Which of the following constitutes a set amphoteric species -

(A) H_3O^+ , $H_2PO_4^-$, HCO_3^-
(B) H_2O , HPO_4^{2-} , $H_2PO_2^-$
(C) H_2O , $H_2PO_3^-$, HPO_4^{2-}
(D) $HC_2O_4^-$, $H_2PO_4^-$, SO_4^{2-}

3. The catalyst used in process of manufacture of H_2 from water gas is -

(A) Finely divided Ni (B) V_2O_5
(C) Pb (D) $Fe_2O_3 + Cr_2O_3$

4. Boron nitride has the structure of the type -

(A) both diamond and graphite
(B) graphite
(C) diamond
(D) NaCl

5. Urea and thiourea exhibits -

(A) Metamerism
(B) Tautomerism
(C) Functional isomerism
(D) Geometrical isomerism

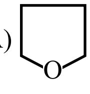
6. I.U.P.A.C. name of $(CH_3)_2CH–CH_2–CH_2Br$ is

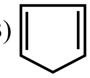
(A) 1-bromo pentane
(B) 2-methyl-4-bromo pentane
(C) 1-bromo-3-methyl butane
(D) 2-methyl-3-bromo propane

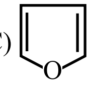
7. Which of the following do not have B–B bond ?

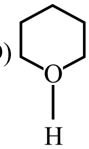
(A) Ni_3B (B) FeB (C) V_3B_2 (D) NaB_{15}

8. Which of the following is heterocyclic aromatic species -

(A) 

(B) 

(C) 

(D) 

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9. The stronger electron withdrawing effect of $\overset{\oplus}{N}R_3$ than NO_2 can be explained on basis of -

(A) Inductive effect

(B) Field effect

(C) Mesomeric effect

(D) Hyper conjugation effect

10. Which of the following is/are correct ?

(A) The boron hydrides having molecular formula B_6H_{11} and B_6H_9 do not exist

(B) In B_4H_{10} there are one B—B bonds

(C) In B_4H_{10} there are four B- - -H - - -B bonds

(D) In B_4H_{10} the B atoms are sp^2 and sp^3 hybridised

11. H_3 PO_3 and H_3BO_3 which of the following is/are correct ?

(A) H_3PO_3 is a dibasic acid and H_3BO_3 is a monobasic acid

(B) In H_3 PO_3 phosphorous is sp^3 hybridised but in H_3 BO_3 boron is sp^2 hybridised

(C) H_3 PO_3 is a Bronsted acid but H_3 BO_3 is a lewis acid

(D) In H_3PO_3 the octet is completed for phosphorous but in H_3 BO_3 the octet is incomplete

12. Which of the following expression is/are true ?

(A) $[H^+] = [OH^-] = \sqrt{K_w}$ for a neutral solution

(B) $[OH^-] < \sqrt{K_w}$ for an acidic solution

(C) pH + pOH = 14 at all temperature

(D) $[OH^-] = 10^{-7}$ M at 25° C

13. Which of the following is/are the soft bases ?

(A) CN^- (B) CO

(C) C_2H_4 (D) H_2O

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage-II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage # 1 (Ques. 14 & 15)

Calcium sulphate is available in nature in two forms, anhydride, $CaSO_4$ and gypsum, $CaSO_4.2H_2O$. Gypsum when heated at 120ºC, from the monoclinic form it changes to orthorhombic form and then it loses three fourth of it's water of crystallisation and plaster of paris is produced

14. What will be the product if gypsum is heated at 200ºC ?

(A) $CaSO_4.\ H_2O$ (B) $2CaSO_4.\ H_2O$

(C) $CaSO_4$ (D) CaO, SO_2 and O_2

15. Suspension of gypsum is used for production of nitrogeneous fertilizer as follows -

(A) $CaSO_4 + NH_3 \longrightarrow (NH_4)_2\ SO_4 + Ca(OH)_2$

(B) $NH_3 + CaSO_4 + CO_2 \longrightarrow (NH_4)_2SO_4 + CaCO_3$

(C) $CO_2 + CaSO_4 \longrightarrow Ca(OH)_2 + CaCO_3$

(D) $NH_3 + CaSO_4 + CO_2 \longrightarrow CaCO_3 + (NH_4)_2CO_3 + H_2SO_4$

Passage # 2 (Ques. 16 to 18)

For general reaction,

$$aA + bB \rightleftharpoons cC + dD$$

equilibrium constant K_c is given by the following relation.

$$K_c = \frac{[C]^c[D]^d}{[A]^a[B]^b}$$

However, when all reactants and products are gases, the equilibrium constant is generally expressed in terms of partial pressures. The relationship between the partial pressure (p) of any one gas in the equilibrium mixture and the molar concentrations can be correlated provided the gas behaves as an ideal gas.

16. The relation between partial pressure of the gas and its molar concentration at a given temperature T is

A) $p = \frac{\text{molar concentration}}{T}$

(B) $p = \frac{\text{molar concentration}}{RT}$

(C) p = molar concentration × RT

(D) p = molar concentration × T

17. Equilibrium constant for the following reaction is

$aA + bB \rightleftharpoons cC + dD$

(A) $K_p = \dfrac{[C]^c[D]^d}{[A]^a[B]^b} \times P$

(B) $K_p = \dfrac{[C]^c[D]^d}{[A]^a[B]^b} \times \dfrac{[RT]^{c-d}}{(RT)^{a-b}}$

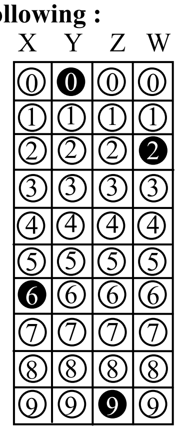
(C) $K_p = \dfrac{[C]^c[D]^d}{[A]^a[B]^b} \times \dfrac{[RT]^{c+d}}{(RT)^{a+b}}$

(D) $K_p = \dfrac{K_c RT}{P}$

18. When 8.1 ml of hydrogen and 9.3 ml of iodine vapour are heated to 444°, 13.5 ml of HI are produced. The equilibrium concentration for H_2 and I_2 respectively should be -

(A) 1.35 and 1.35 (B) 2.55 and 2.55
(C) 1.35 and 2.55 (D) 2.55 and 1.35

This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



19. Consider the reaction $AB_{2(g)} \rightleftharpoons AB_g + B_{(g)}$. It the initial pressure of AB_2 is 100 torr and equilibrium pressure is 120 torr. The equilibrium constant Kp in terms of torr is.

20. It 50 ml of 0.2 M NaCN is mixed with 50 ml of 0.2 M HCl then (K_b for CN^- = 2 × 10^{-5}) Calculate concentration of $[H_3O^+]$ interms of molarity × 10^{-6}.

21. On heating CaC_2O_4 , ………type of gases are produced

22. In borax ($Na_2B_4O_7$, $10H_2O$) the number of B-O-B bonds is ………

23. The basicity of boric acid is ……..

24. In H_2O (s) one H_2O molecule is surrounded by ……… other H_2O molecules by hydrogen bonds.

25. The total number of Acyclic isomers possible for C_3H_6O is/are …………

26. How many structural isomers are possible with molecular formula C_3H_6O ?

27. C_7H_8O shows how many Isomers

MATHEMATICS

Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. The locus of point P(x, y) such that $\sqrt{x^2+y^2+8y+16} - \sqrt{x^2+y^2-6x+9} = 5$ is -

(A) a circle (B) a finite line segment
(C) a parabola (D) part of line $\dfrac{x}{3} - \dfrac{y}{4} = 1$

2. Let lines $(y - 2) = m_1(x - 3)$ & $y + 4 = m_2(x - 5)$ intersect at right angles at P (where m_1, m_2 are parameters). If locus of P is a circle $x^2 + y^2 + gx + fy + c = 0$, then $f - g + c$ equals -

(A) 10 (B) 13 (C) 17 (D) 22

3. A variable line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ slides on the coordinate axes in second quadrant such that sum of length of intercepts made by it on axes is ℓ, then the locus of circumcentre of triangle made by the line with the axes is -

(A) $2(x - y) + \ell = 0$ (B) $2(x + y) - \ell = 0$
(C) $2(x + y) + \ell = 0$ (D) None

4. If acute angle between the lines represented by $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$ is $\tan^{-1} m$ and $a^2 + b^2 - ab - a - b + 1 \le 0$ then $3a + 2b$ equals -

(A) $\dfrac{1}{2m}$ (B) $\dfrac{1}{m}$ (C) m (D) 2m

5. A rhombus ABCD has sides of length 8 cm. A circle with centre 'A' passes through C (opposite vertex). Also a circle with centre B passes through D. If two circles are tangent to each other, then area of rhombus –

(A) 75 sq. units (B) 48 sq. units
(C) 30 sq. units (D) None

6. If length of focal chord of $y^2 = 4ax$ is ℓ, then angle between axis of parabola and focal chord is -

(A) $\pm \sin^{-1}\sqrt{\frac{2a}{\ell}}$ (B) $\pm \sin^{-1}\sqrt{\frac{4a}{\ell}}$

(C) $\pm \tan^{-1}\sqrt{\frac{4a}{\ell}}$ (D) None of these

7. If $\vec{b}$ and $\vec{c}$ are two non collinear vectors such that $\vec{a}\cdot(\vec{b}+\vec{c}) = 4$ and $\vec{a}\times(\vec{b}\times\vec{c}) = (x^2 - 2x + 6)\,\vec{b} + (\sin y)\,\vec{c}$, then the point (x, y) always lies on -

(A) $x = 1$ (B) $y = 1$

(C) $y = \pi$ (D) $x + y = 0$

8. If a, b, c are the p^{th}, q^{th}, r^{th}, terms of an A.P. respectively and $\vec{x} = (q - r)\hat{i} + (r - p)\hat{j} + (p - q)\hat{k}$ & $\vec{y} = a\,\hat{i} + b\,\hat{j} + c\,\hat{k}$ then -

(A) $\vec{x}, \vec{y}$ are parallel vectors

(B) $\vec{x}\times\vec{y} = \hat{i} + \hat{j} + \hat{k}$

(C) $\vec{x}\cdot\vec{y} = 1$

(D) $\vec{x}, \vec{y}$ are orthogonal vectors

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. Tangents are drawn to the circle $x^2 + y^2 = 32$ from a point A lying on x-axis. The tangent cuts y axis at B & C points. Then point A such that area of ΔABC is minimum can be -

(A) (6, 0) (B) (–8, 0)

(C) (–6, 0) (D) (8, 0)

10. If two distinct chords of a parabola $y^2 = 4ax$, passing through $(a, 2a)$ are bisected on line $x + y = 1$, then length of latus rectum can be-

(A) 1 (B) 2 (C) 3 (D) 4

11. If $\vec{a}\times\vec{b} = \vec{c}$ and $\vec{b}\times\vec{c} = \vec{a}$, then which of the following is/are correct -

(A) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs

(B) $\vec{b}$ is a unit vector

(C) $|\vec{a}| = |\vec{c}|$

(D) $\vec{a}, \vec{b}, \vec{c}$ are not orthogonal to each other

12. If a vector $\vec{r}$ of magnitude $3\sqrt{6}$ is directed along bisector of the angle between the vector $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ & $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$, then $\vec{r}$ is equal to -

(A) $\hat{i} - 7\hat{j} + 2\hat{k}$ (B) $\hat{i} + 7\hat{j} - 2\hat{k}$

(C) $-\hat{i} + 7\hat{j} - 2\hat{k}$ (D) $\hat{i} - 7\hat{j} - 2\hat{k}$

13. Equation of a plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is -

(A) 4x + 3y + 5z = 25 (B) 4x + 3y + 5z = 50

(C) 3x + 4y + 5z = 49 (D) x + 7y – 5z = 2

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage-II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage # 1 (Ques. 14 & 15)

The equation of a curve is
$f(x,y) = 9x^2 - 24xy + 16y^2 - 20x - 15y - 60 = 0$

14. Equation of axis of curve is -

(A) $3x = y$ (B) $3x + 4y = 0$

(C) $3x + y = 0$ (D) $3x - 4y = 0$

15. Equation of directrix is

(A) $16x + 9y = 53$ (B) $16x + 12y + 53 = 0$

(C) $16x + 2y = 53$ (D) None

Passage # 2 (Ques. 16 to 18)

Consider a plane having equation $\vec{r}\cdot\vec{n} = d$ (where $\vec{n}$ should not be unit vector) & two points A($\vec{a}$) & B($\vec{b}$) are lying on same side w.r.t. the plane.

16. If foot of perpendiculars from A & B to the plane are A′ & B′ respectively then distance A'B' is equal to -

(A) $\frac{|(\vec{b}-\vec{a})\cdot\vec{n}|}{|\vec{n}|}$ (B) $|(\vec{b}-\vec{a})\cdot\vec{n}|$

(C) $\frac{|(\vec{b}-\vec{a})\times\vec{n}|}{|\vec{n}|}$ (D) $|(\vec{b}-\vec{a})\times\vec{n}|$

17. Reflection of the A($\vec{a}$) w.r.t. the plane has the position vector -

(A) $\vec{a} + \frac{2}{n^2}(d - \vec{a}\,.\,\vec{n})\,\vec{n}$ (B) $\vec{a} + \left(\frac{d - \vec{a}\,.\,\vec{n}}{n^2}\right)\vec{n}$

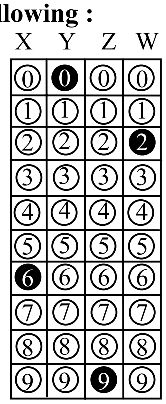
(C) $\vec{a} + \frac{2}{n^2}(d + \vec{a}\,.\,\vec{n})\,\vec{n}$ (D) none of these

18. If a plane is drawn from the point $\vec{a}$ parallel to $\vec{r}\,.\,\vec{n} = d$ & another plane is drawn from the point $\vec{b}$ parallel to $\vec{r}\,.\,\vec{n} = d$ and the distance between two planes is d_1 then $(A'\,B')^2 + d_1^2$, is equal to-

(A) $\frac{(\vec{b} - \vec{a})^2}{n^2}$ (B) $|\vec{b} - \vec{a}|^2\, n^2$

(C) $(\vec{b} - \vec{a})^2$ (D) n^2

This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



19. Number of straight lines equidistant from three non collinear points in the plane of points is.

20. If x & y are non zero real no. satisfying $xy\,(x^2 - y^2) = (x^2 + y^2)$ then minimum value of $x^2 + y^2$ is.

21. The slope of the line which belongs to family of line $(1 + \lambda)\,x + (\lambda - 1)\,y + 2(1 - \lambda) = 0$ and makes shortest intercept on $x^2 = 4y - 4$ is………….

22. The number of points on hyperbola $xy = c^2$ from which two tangents drawn to ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $b < a < c$) are perpendicular to each other is………….

23. Two circle of radii 'a' and 'b' touching externally are inscribed in area bounded by $y = \sqrt{1 - x^2}$ and x-axis. If $b = \frac{1}{2}$ and $a = \frac{1}{k}$, then k is........

24. If $\overrightarrow{AB}\,.\,\overrightarrow{CD} = \frac{k}{2}(\,|\overrightarrow{AD}|^2 + |\overrightarrow{BC}|^2 - |\overrightarrow{AC}|^2 - |\overrightarrow{BD}|^2)$, then the value of k is.

25. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 1$, angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{3}$ and $\vec{c}$ satisfies $2(\vec{a} + \vec{b}) - \vec{c} = \vec{b} \times \vec{c}$, then the value of $|(\vec{a} \times \vec{c})\,.\,\vec{b}|$ is.

26. In a regular tetrahedron let θ be the angle between any edge and a face not containing the edge. If $\cos^2\theta = \frac{a}{b}$ where $a, b \in I^+$ also a and b are coprime, then find the value of $\frac{5}{13}(10a + b)$

27. Let A (1, 2), B (3, 4) be two point and C (x,y) be a point such that $(x - 1)(x - 3) + (y - 2)(y - 4) = 0$. If area of ΔABC is 1 sq, unit. Then maximum number of positions of C in xy plane is.

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1. An element's name must be approved by the International Union of Pure and Applied Chemistry, or I.U.P.A.C., in Geneva, Switzerland.
2. The heaviest element gas is radon at room temperature. (There may be heavier ones, but they are compunds not atoms). It was discovered by Friedrich Ernst Dorn in Germany in 1900, but he first called it niton, until 1923.
3. The lightest gas is hydrogen, it is also the lightest of all elements.
4. The element with the highest melting/freezing point is carbon at 6,381 degrees Fahrenheit (3,527 degrees Celsius).
5. The element with the highest boiling point is rhenium at 10,105 degrees Fahrenheit (5,596 degrees Celsius).

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Ans	B,C	A,B,C	A,C	B,C	A	B	D	A	A
Numerical	19	20	21	22	23	24	25	26	27
Response	5	3	6	3	1	8	4	5	8

CHEMISTRY

Ques	1	2	3	4	5	6	7	8	9
Ans	C	A	B	C	C	B	D	D	A,B,C
Ques	10	11	12	13	14	15	16	17	18
Ans	A,B,C	A,B,D	B,C,D	C,D	A	B	A	D	D
Numerical	19	20	21	22	23	24	25	26	27
Response	3	4	9	3	3	4	3	9	1

MATHEMATICS

Ques	1	2	3	4	5	6	7	8	9
Ans	D	C	A	B	A	D	D	D	A,C
Ques	10	11	12	13	14	15	16	17	18
Ans	A,B	A,C,D	B,D	C	D	A	C	B	D
Numerical	19	20	21	22	23	24	25	26	27
Response	2	4	4	2	4	0	9	9	5

IIT- JEE 2013 (July issue)

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Ans	D	C	C	A	B	D	D	C	A,C,D
Ques	10	11	12	13	14	15	16	17	18
Ans	A,D	A,B,C	A,C,D	A,B	C	D	B	D	B
Numerical	19	20	21	22	23	24	25	26	27
Response	4	1	5	1	5	5	0	2	4

CHEMISTRY

Ques	1	2	3	4	5	6	7	8	9
Ans	D	C	D	B	B	C	A	C	A,B
Ques	10	11	12	13	14	15	16	17	18
Ans	A,B,C	A,B,C,D	A,B,D	A,B,C	C	B	C	C	C
Numerical	19	20	21	22	23	24	25	26	27
Response	5	7	1	5	1	2	6	9	5

MATHEMATICS

Ques	1	2	3	4	5	6	7	8	9
Ans	D	C	A	B	B	B	A	D	B,D
Ques	10	11	12	13	14	15	16	17	18
Ans	A,B,C	A,B,C	A,C	B	D	B	C	A	C
Numerical	19	20	21	22	23	24	25	26	27
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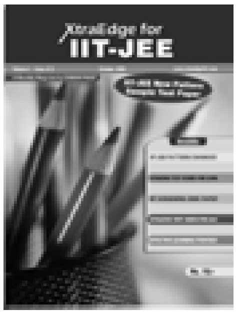

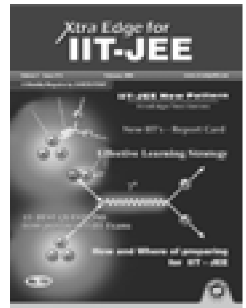

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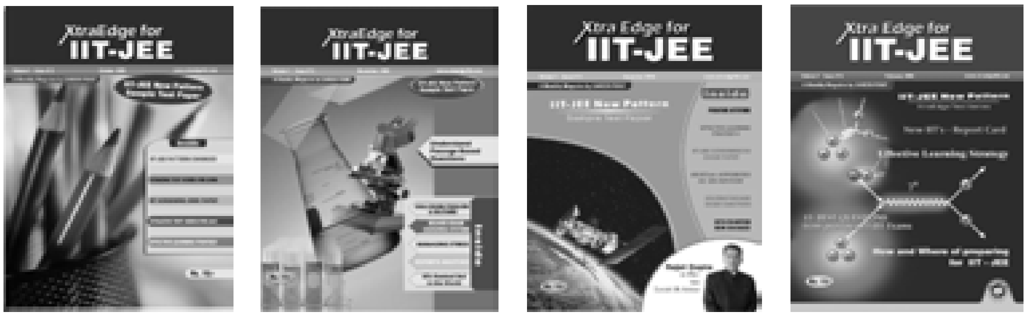

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